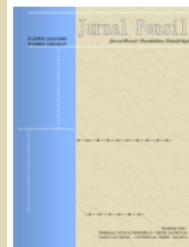


Available online at: <http://journal.unj.ac.id>

Jurnal
Pensil Pendidikan Teknik Sipil

Journal homepage: <http://journal.unj.ac.id/unj/index.php/jpensil/index>



CORRELATION OF EFFECTIVE INERTIA OF CASTELLATED BEAMS WITH SPAN-TO-DEPTH RATIO

Sutedjo Krisnadi^{1*}, *Ignatius Sudarsono*², *Rully Savitri Nurvita*³, *Nadiv Raka Pradifa*⁴
^{1,2,3,4} Program Studi Teknik Sipil, Fakultas Teknik, Universitas Langlangbuana
Jalan Karapitan 116, Bandung, Jawa Barat, 40261, Indonesia
^{*1}sutedjokrisnadi88@gmail.com, ²Ignazsd2@gmail.com, ³rullysavitrin@gmail.com,
⁴nadiv.pradifa9@gmail.com

Abstract

Material efficiency in structural systems can be enhanced through cross-sectional optimization of structural elements, one of which is the use of castellated steel beams. A castellated beam has an increased section depth without adding material volume from the original beam. Although several studies have investigated the flexural performance of castellated beams, analytical studies on the moment of inertia, particularly those considering the effect of shear deformation, remain limited. This study compares the net moment of inertia (I_{nett}) based on the AISC Design Guide 31 with the effective moment of inertia (I_{eff}) obtained from Finite Element Analysis (FEA). Beam models were developed with total depths (d_g) ranging from 450 mm to 1350 mm and span lengths (L) from 6000 mm to 18000 mm. The I_{eff} values were derived from the midspan deflection under uniformly distributed loading and correlated with the L/d_g ratio. Results show that I_{eff} increases with the increasing L/d_g and tends to converge. Empirical correlation between L/d_g and $I_{\text{nett}}/I_{\text{eff}}$ was obtained with a coefficient of determination of $R^2 = 0.9412$. When $L/d_g \geq 18$, the $I_{\text{nett}}/I_{\text{eff}}$ ratio is less than 1.1, indicating that the difference between I_{nett} and I_{eff} is below 10%. Therefore, the I_{nett} can be safely used for structural design. The findings provide a practical guideline for estimating the effective flexural stiffness of castellated beams and contribute to the development of analytical and numerical approaches for their structural design.

P-ISSN: [2301-8437](#)

E-ISSN: [2623-1085](#)

ARTICLE HISTORY

Accepted:

20 Maret 2026

Revision:

26 Mei 2026

Published:

31 Mei 2026

ARTICLE DOI:

[10.21009/jpensil.v15i2.66798](https://doi.org/10.21009/jpensil.v15i2.66798)



Jurnal Pensil :
Pendidikan Teknik
Sipil is licensed under a
[Creative Commons
Attribution-ShareAlike
4.0 International License](#)
(CC BY-SA 4.0).

Keywords: Castellated Steel Beam, Moment of Inertia, Finite Element Analysis, Shear Deformation, L/d_g Ratio

Introduction

Material efficiency is one of the key factors in achieving an optimum structural design in terms of both cost and constructability. One of the most widely applied innovations to achieve such efficiency is the use of castellated steel beams (Kowsalya & Iyappan, 2020), which are engineered from wide-flange beams that are cut according to a specific pattern and then rejoined in a zigzag configuration. This process produces a new beam with a sectional depth approximately 1.5 times greater than that of the original member, and with a series of hexagonal openings along the web, resulting in increased bending stiffness without a significant increase in self-weight (Elsa Sabu & Joseph, 2022; Fares et al., 2016).

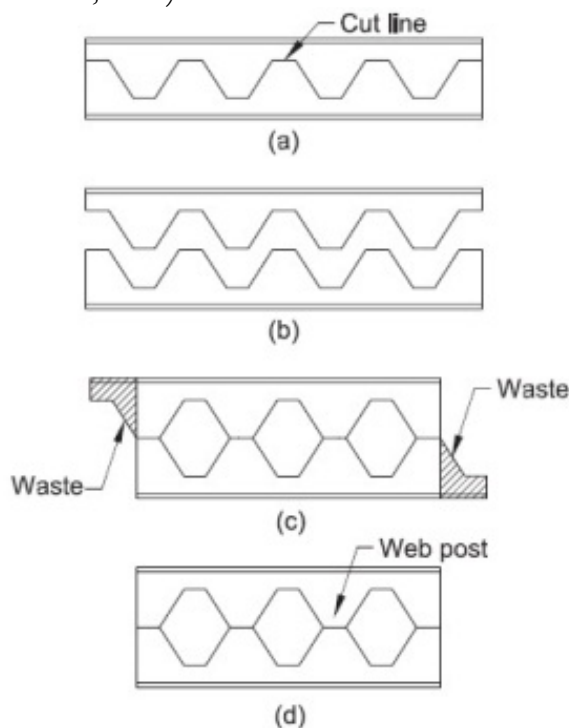


Figure 1. Castellation beam fabrication process (Fares et al., 2016)

Figure 1 illustrates the fabrication process of a castellated steel beam, in which the cut portions of the parent beam are re-welded using full-penetration welding. According to Zhou et al. (2018), welding at the web-post region does not produce residual stresses greater than those in the original beam. Previous studies have also shown that castellated beams exhibit significantly improved flexural performance compared with their parent wide-flange beams, with a higher strength-to-weight ratio, making them a more efficient option for structural steel applications (Elsa Sabu & Joseph, 2022; Shaikh & Autade, 2016). Yustisia et al. (2020) compared different opening configurations, while Abhale and Wakchaure (2017) evaluated castellated beam designs based on several design codes.

According to Elsa Sabu and Joseph (2022), castellated beams demonstrate a substantial increase in flexural performance relative to their parent beams. Because the material composition remains identical, this enhancement primarily results from geometric efficiency, leading to a higher strength-to-weight ratio. Consequently, castellated beams are considered structurally more efficient than conventional wide-flange sections. Wakchaure et al. (2012) reported that the optimum strength enhancement occurs when the opening height is approximately 0.6 times the original beam depth (d) with an opening angle of 60° . Similar findings were presented by Budi et al. (2017), who observed maximum flexural strength at a 60° hexagonal opening, while Barkiah and Darmawan (2021) identified an optimum angle of 50° under comparable conditions.

The flexural capacity enhancement from the parent beam to the castellated beam is generally achieved through the increase in overall beam depth, which directly increases the moment of inertia. Numerous studies have attempted to further improve flexural performance through various geometric and connection modifications. Al-Thabthawee and Al-Kannoon (2018) and Zarmiham et al. (2023a, 2023b) investigated the use of web-post spacers to increase the effective beam depth, transforming the openings into an octagonal shape. Upadhyay et al. (2021) and Mehetre et al. (2020) proposed curved-edge openings to reduce stress concentrations at the sharp corners of hexagonal holes. Meanwhile, Liu et al. (2020) evaluated bolted web-post connections as an alternative to welded joints to mitigate residual stresses, and Hoseinpour et al. (2018) studied the effect of lateral stiffeners in preventing lateral-torsional buckling. Although various approaches have been proposed, flexural improvement fundamentally depends on the increase in moment of inertia resulting from the enlarged beam depth (Weidlich et al., 2021). However, this increase also amplifies the shear-deformation contribution to the overall beam stiffness, as reported by Deepha et al. (2020) and Deepha & Jayalekshmi (2020). The studies on shear contribution have also been developed by Kumar et al. (2015), Wang et al. (2016), and Najafi and Wang (2017).

From a theoretical perspective, increasing the depth of a castellated beam section enhances its flexural capacity. However, the beam flexural resistance to the minor axis becomes weaker compared to that about the major axis, which may lead to lateral-torsional buckling. This phenomenon has been investigated by Kim et al. (2016), Sonck and Bellis (2016, 2017), Gunawan and Suryoatmono (2017), Kwani and Wijaya (2017), Subramani and Sukumar (2018), Hadeed and Alshimmeri (2019), Sandeep et al. (2020), Huang and Cao (2025), and many others. Nevertheless, the lateral-torsional buckling is not considered in this study because the applied loads are relatively small and insufficient to trigger that effect.

The moment of inertia is a fundamental parameter that governs the flexural stiffness and the ability of structural members to resist transverse loading. A larger moment of inertia produces smaller deflections under identical loading conditions. In castellated beams, geometric modifications due to cutting and re-welding of the web significantly change the sectional mass distribution, thereby affecting the actual moment of inertia. In addition, the reduction of effective web area increases shear deformation, which may reduce total stiffness if not properly considered. Therefore, a simultaneous evaluation of effective moment of inertia and shear contribution is required to model structural behavior more accurately.

Although numerous studies have highlighted the flexural strength enhancement caused by geometric modifications in castellated beams, research specifically addressing the variation of moment of inertia due to different opening shapes and expansion ratios remains limited. Most previous works focused on flexural capacity improvement through experimental or numerical studies, but few have provided analytical formulations capable of accurately predicting the change in moment of inertia for various opening configurations. In moment resisting frame systems, however, the accuracy of moment of inertia values is essential for predicting moment distribution, joint rotation, and lateral deflection control. Hence, this study focuses on examining the effects of expansion ratio and opening pattern on the moment of inertia of castellated beams through analytical modeling and finite element analysis (FEA), aiming to develop a parametric relationship that can serve as a practical and accurate basis for structural design. Moreover, within moment resisting frame analysis, the influence of axial load on beam behavior is relatively insignificant compared to the dominant lateral load that governs flexural deformation (El-Tobgy et al., 2021).

Finite element analysis is adopted as the primary analytical tool in this research because it can provide highly accurate and realistic results when the modeling approach and parameters are properly defined. Qiao et al. (2022) conducted experimental tests to validate FEA modeling using ABAQUS and demonstrated that finite element simulations could effectively represent the flexural behavior of castellated beams. Similarly, Anbarasu et al. (2021) investigated box-section steel beams modified into castellated box beams, where FEA results showed excellent agreement with experimental findings. Other validation studies have been reported by Frans et al. (2017),

Morkhade and Gupta (2017), Elaiwi et al. (2019a, 2019b), Serene and Aswathy (2019), Haris et al. (2023), Maali and Cinar (2024), and Shiyekar et al. (2024). These studies confirm the reliability of numerical methods based on FEA as an effective approach for analyzing the influence of geometric parameters and expansion ratios on the moment of inertia of castellated beams.

The novelty of this study lies in establishing a quantitative relationship between the span-to-depth ratio (L/d_g) and the ratio between the net moment of inertia (I_{nett}) from AISC Design Guide 31 and the effective moment of inertia (I_{eff}) obtained from finite element analysis. This relationship provides a practical criterion for identifying the geometric limit at which shear deformation effects become negligible and the net moment of inertia can be safely used for structural design.

Research Methods

A numerical research design with a comparative analytical approach was implemented to evaluate the moment of inertia of castellated steel beams. The study focused on castellated beam models derived from wide-flange steel sections featuring hexagonal openings. The net moment of inertia (I_{nett}) was determined in accordance with AISC Design Guide 31. Finite Element Analysis (FEA) was used to obtain the effective moment of inertia (I_{eff}), which incorporates the contribution of the remaining cross-sectional elements and the effects of shear deformation. The results were compared to assess the relationship and differences between I_{nett} and I_{eff} as measures of the effective flexural stiffness of castellated beams and the impact of shear deformation.

Research Results and Discussion

The comparison between the effective moment of inertia (I_{eff}) and the net moment of inertia (I_{nett}) is carried out to evaluate the validity of using I_{nett} in structural design practices. The value of I_{eff} obtained from finite element analysis is influenced by shear deformation occurring in the web of the beam. As the beam span increases, the influence of shear deformation to the flexural stiffness becomes smaller, causing I_{eff} to converge toward its true value. The ratio of beam span to castellated beam depth (L/d_g) is used as the parameter to assess the magnitude of shear deformation effects on beam stiffness and to identify the condition where the shear deformation can be neglected.

Theoretical Moment of Inertia (I_{nett})

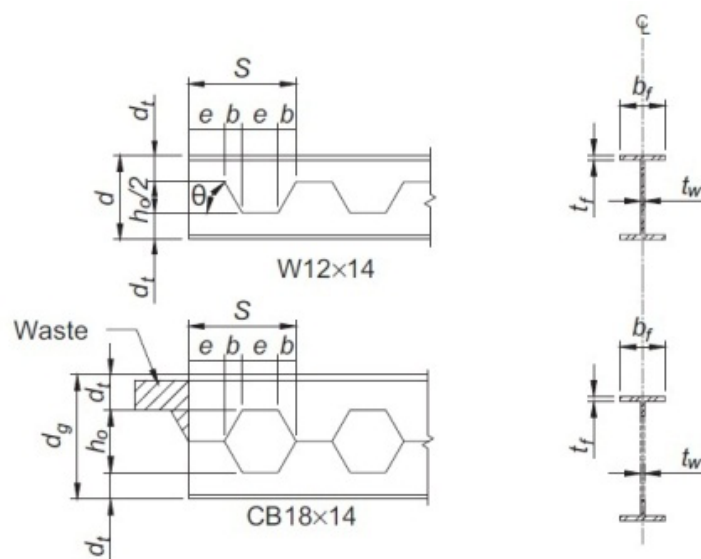


Figure 2. Transformation of WF12x14 into castellated beam CB18x14 (Fares et al., 2016)

Figure 2 shows the transformation of a W12×14 wide flange (WF) into a CB18×14 castellated beam, with a final depth (d_g) equal to 1.5 times the original beam depth (d). The castellated beam contains hexagonal openings with an inclination angle (Q), which is taken as 60° in this study. The cross-section through a hexagonal opening consists of two T-shaped sections located at the top and bottom flanges, with T-section height d_t and opening height h_o . The distance between the centroids of the two T-sections is defined as the effective depth (d_{eff}) of the castellated beam section.

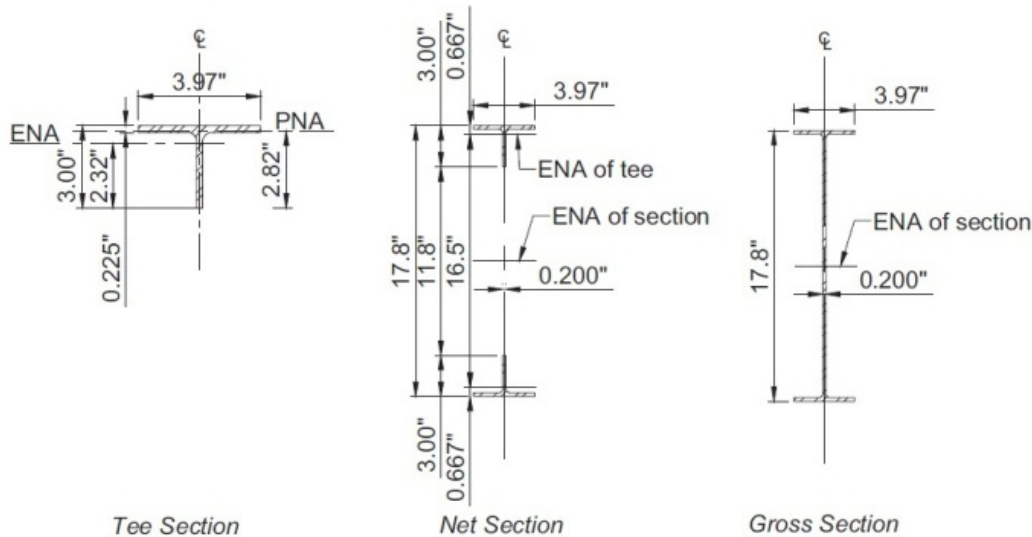


Figure 3. Gross and net sections of CB18x14 (Fares et al., 2016)

Figure 3 shows a comparison between the net section and the gross section of the CB18×14 castellated beam. The net section represents the remaining area after the reduction of the hexagonal openings, while the gross section is the complete geometry of the beam. The net moment of inertia (I_{net}) is obtained from the net section using the parallel axis theorem.

The procedure for determining I_{net} is as follows:

1. Determine the dimensions of the parent wide flange section used to form the castellated beams, namely WF300, WF400, WF500, WF600, WF700, WF800, and WF900.
2. Define the target final depth of the castellated beam (d_g), namely 450 mm, 600 mm, 750 mm, 900 mm, 1050 mm, 1200 mm, and 1350 mm.
3. Compute the moment of inertia of the upper and lower T-sections forming the net section.
4. Determine the centroidal distance of each T-section relative to the overall neutral axis of the beam.
5. Calculate I_{net} using the following equation:

$$I_{net} = \sum I_i + \sum A_i \cdot d_i^2$$

Where:

- I_{net} = net moment of inertia (mm^4)
- I_i = T-section moment of inertia (mm^4)
- A_i = T-section sectional area (mm^2)
- d_i = T-sections centroidal distance (mm)

The value of the I_{net} represents the theoretical moment of inertia according to the AISC Design Guide 31, which is typically used in structural design practices. The results are then compared with the effective moment of inertia (I_{eff}) obtained from the finite-element analysis.

Table 1 shows the wide-flange (WF) beams used as the base sections in this study. They have the original height section (d), flange width (b_f), flange thickness (t_f), and web thickness (t_w) as listed in Table 1.

Table 1. Wide-flange sections used in the study

Wide Flange Beams	d [mm]	b_f [mm]	t_f [mm]	t_w [mm]
WF300x300	300	300	15	10
WF400x300	390	300	16	10
WF500x300	488	300	18	11
WF600x300	588	300	20	12
WF700x300	700	300	24	13
WF800x300	800	300	26	14
WF900x300	900	300	28	16

Table 2 shows the castellated beams formed sequentially from their corresponding WF parent beams. Each castellated beam has a total depth (d_g) and an effective depth (d_{eff}) determined in accordance with the AISC Design Guide. The flange width, flange thickness, and web thickness of the castellated beams remain identical to those of the original WF beams.

Table 2. Castellated beam sections derived from wide-flange sections

Wide Flange Beams	Castellated Beams	d_g [mm]	d_{eff} [mm]	I_{nett} [x 10 ⁹ mm ⁴]
WF300x300	CB450x300	450	426.18	0.4637
WF400x300	CB600x300	585	554.85	0.8654
WF500x300	CB750x300	732	692.67	1.5722
WF600x300	CB900x300	882	832.22	2.6101
WF700x300	CB1050x300	1050	988.51	4.4850
WF800x300	CB1200x300	1200	1126.40	6.5068
WF900x300	CB1350x300	1350	1260.61	9.2003

The net moment of inertia (I_{nett}) of each castellated beam was calculated using Equation (1), which refers to the net cross-section formed by the two T-shaped components. The value of the I_{nett} for each beam section is constant and depends solely on the geometric configuration and sectional dimensions, without considering the influence of shear deformation or the span to depth ratio (L/d_g). The theoretical values of I_{nett} as shown in Table 2 serve as the reference for evaluating the effective moment of inertia (I_{eff}) obtained from the finite element analysis in the subsequent section.

Finite Element Analysis (I_{eff}) and Its Comparison with I_{nett}

Numerical analysis was performed to obtain the effective moment of inertia (I_{eff}) of castellated steel beams through modeling using Finite Element Analysis (FEA). The moment of inertia was not obtained directly from the FEA output but was calculated using the elastic deflection equation for beams. Finite element modeling was employed because it has been widely proven to provide accurate results for analyzing perforated steel structures (Anbarasu et al., 2021; Qiao et al., 2022). The software used in this study was ANSYS Workbench 2020 R1.

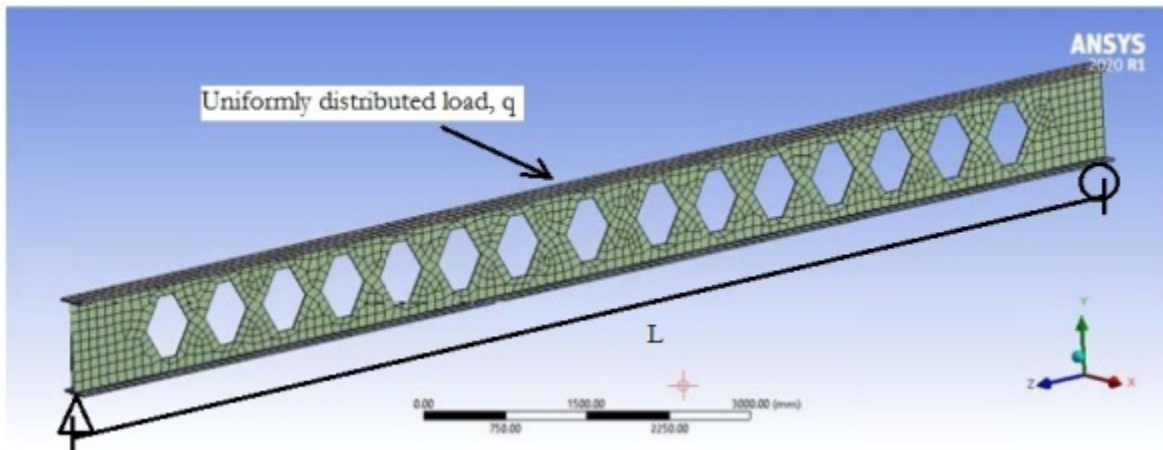


Figure 4. Discretization of castellated beam models

The castellated beam models were developed with sectional depths (d_g) of 450 mm, 600 mm, 750 mm, 900 mm, 1050 mm, 1200 mm, and 1350 mm. The beam spans (L) were defined as 6000 mm, 9000 mm, 12 000 mm, 15 000 mm, and 18 000 mm. Each beam was modeled with simply supported end conditions—pinned at one end and roller-supported at the other—without any lateral bracing, as illustrated in Figure 4. A uniformly distributed load of $q = 30 \text{ N/mm}$ was applied along the entire span, and the maximum mid-span deflection was recorded. This configuration represents a typical condition of beams in moment-resisting frame systems, except that axial force effects were not considered. A similar configuration of simply supported castellated beams under uniformly distributed loads was also investigated by Yuan (2016).

Each castellated beam was discretized using 20-node brick solid elements. A deflection-based mesh convergence study was performed prior to the analysis, and an element size of 50 mm was selected as the optimum mesh size. The steel material was defined as an elasto-plastic, with an elastic modulus of $E = 2 \times 10^5 \text{ MPa}$, Poisson's ratio $\nu = 0.3$, and a nominal yield strength $F_y = 240 \text{ MPa}$. However, the analysis was performed under linear-elastic conditions, so material plasticity and geometric nonlinearity were neglected.

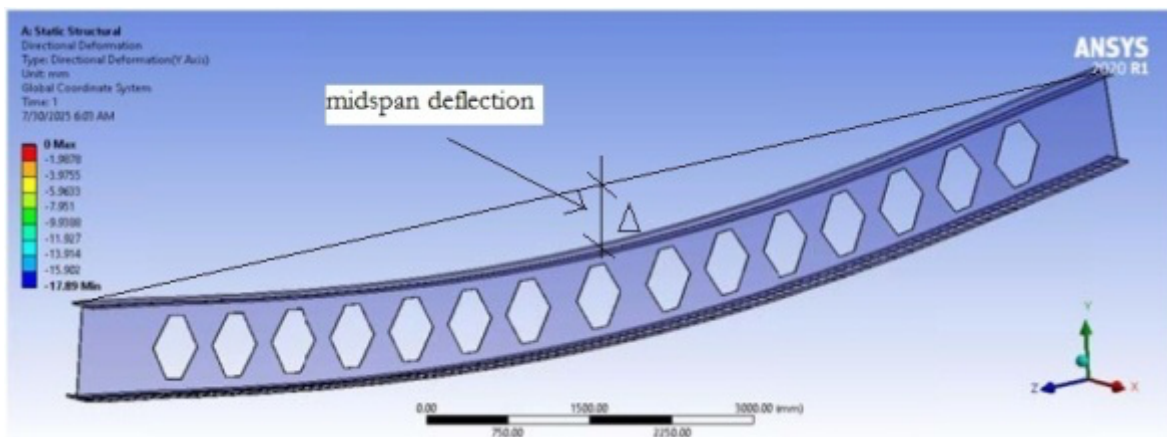


Figure 5. Midspan deflection of castellated beam models obtained from FEA

The primary output of the numerical simulation was the maximum mid-span deflection (Δ), as illustrated in Figure 5. Based on classical beam theory for a simply supported beam under uniform load, the relationship between maximum deflection and the moment of inertia, according to Wang (1983), can be expressed as:

$$I_{\text{eff}} = \frac{5 \cdot q \cdot L^4}{384 \cdot E \cdot \Delta}$$

Where:

- I_{eff} = effective moment of inertia of the castellated beam (mm^4)
- q = uniformly distributed load (N/mm)
- L = beam span (mm)
- E = modulus of elasticity ($2 \times 10^5 \text{ MPa}$)
- Δ = mid-span deflection (mm)

The calculated I_{eff} represents the actual flexural stiffness of the castellated beam, which includes the effects of shear deformation and the combined contribution of both net and gross sectional areas. The total mid-span deflection is the sum of the bending deflection and the shear deformation component, expressed as:

$$\Delta_{\text{total}} = \Delta_{\text{b}} + \Delta_{\text{s}}$$

Where:

- Δ_{total} = total deflection of the beam (mm)
- Δ_{b} = deflection due to bending (mm)
- Δ_{s} = deflection due to shear deformations (mm)

This relationship indicates that the value of I_{eff} analyzed by the FEA and elastic beam formulation inherently accounts for the influence of shear deformation on the overall flexural stiffness of the castellated beam.

The numerical analysis using Finite Element Analysis (FEA) produced the maximum midspan deflection for each castellated beam geometry variation. Based on these results, the effective moment of inertia (I_{eff}) was calculated using Equation (2).

Table 3 presents the maximum midspan deflection (Δ) and the corresponding I_{eff} values obtained for castellated beams CB450×300, CB600×300, CB750×300, CB900×300, CB1050×300, CB1200×300, and CB1350×300, each analyzed under five different span lengths (L). The table shows that the I_{eff} values increase as the beam span length increases and tend to converge beyond a certain limit. This indicates that as the ratio between beam span and section depth becomes larger, the effect of shear deformation on flexural stiffness diminishes and can be neglected. Accordingly, the I_{eff} values obtained under such conditions can be regarded as representing the true stiffness of the castellated beam.

The ratios L/d_g and $I_{\text{nett}}/I_{\text{eff}}$, where the parameters d_g and I_{nett} for each castellated beam section are listed in Table 2, were used to investigate this phenomenon. The analysis shows that as the L/d_g ratio increases, the $I_{\text{nett}}/I_{\text{eff}}$ ratio gradually approaches unity, indicating that the difference between I_{nett} and I_{eff} decreases. This suggests that the conventional net moment of inertia (I_{nett}) used in structural design provides a sufficiently accurate approximation of the effective moment of inertia (I_{eff}) within a certain L/d_g range. In this study, when the difference between I_{nett} and I_{eff} is less than 10%, or equivalently when the $I_{\text{nett}}/I_{\text{eff}}$ ratio is less than 1.1, the value of I_{nett} can be considered approximately equal to I_{eff} and therefore acceptable for structural design applications. Overall, the results indicate that I_{eff} approaches I_{nett} as the L/d_g ratio increases, confirming that the influence of shear deformation on flexural stiffness becomes less significant. This pattern serves as the basis for the subsequent analysis, in which the empirical relationship between the L/d_g ratio and the $I_{\text{nett}}/I_{\text{eff}}$ ratio is investigated to determine the critical limit beyond which shear deformation can be neglected and I_{nett} can be considered equivalent to the value of I_{eff} .

Table 3. Effective moment of inertia (I_{eff}) of castellated beams

Castellated Beams	Beam Span	Ratio of L/d_g	Midspan deflection Δ	I_{eff} (equation 2)	I_{nett} (Table 1)	Ratio of I_{nett}/I_{eff}
	[mm]		[mm]	[$\times 10^9 \text{ mm}^4$]	[$\times 10^9 \text{ mm}^4$]	
CB450x300	6000	13.33	7.00	0.3614	0.4637	1.28
	9000	20.00	31.05	0.4128	0.4637	1.12
	12000	26.67	92.96	0.4357	0.4637	1.06
	15000	33.33	221.06	0.4473	0.4637	1.04
	18000	40.00	451.53	0.4541	0.4637	1.02
CB600x300	6000	10.26	4.07	0.6219	0.8654	1.39
	9000	15.38	17.33	0.7397	0.8654	1.17
	12000	20.51	50.95	0.7949	0.8654	1.09
	15000	25.64	120.17	0.8228	0.8654	1.05
	18000	30.77	243.72	0.8413	0.8654	1.03
CB750x300	6000	8.20	2.46	1.0291	1.5722	1.53
	9000	12.30	10.03	1.2776	1.5722	1.23
	12000	16.39	29.01	1.3960	1.5722	1.13
	15000	20.49	67.67	1.4611	1.5722	1.08
	18000	24.59	136.43	1.5028	1.5722	1.05
CB900x300	6000	6.80	1.58	1.6025	2.6101	1.63
	9000	10.20	6.26	2.0469	2.6101	1.28
	12000	13.61	17.89	2.2638	2.6101	1.15
	15000	17.01	41.33	2.3923	2.6101	1.09
	18000	20.41	83.31	2.4610	2.6101	1.06
CB1050x300	6000	5.71	1.02	2.4904	4.4850	1.80
	9000	8.57	3.92	3.2682	4.4850	1.37
	12000	11.43	10.91	3.7122	4.4850	1.21
	15000	14.29	24.91	3.9697	4.4850	1.13
	18000	17.14	49.64	4.1307	4.4850	1.09
CB1200x300	6000	5.00	0.77	3.2842	6.5068	1.98
	9000	7.50	2.81	4.5637	6.5068	1.43
	12000	10.00	7.75	5.2261	6.5068	1.25
	15000	12.50	17.56	5.6321	6.5068	1.16
	18000	15.00	34.67	5.9133	6.5068	1.10
CB1350x300	6000	4.44	0.54	4.6628	9.2003	1.97
	9000	6.67	2.08	6.1724	9.2003	1.49
	12000	8.89	5.58	7.2643	9.2003	1.27
	15000	11.11	12.59	7.8530	9.2003	1.17
	18000	13.33	24.76	8.2801	9.2003	1.11

Correlation between L/d_g Ratio and I_{nett}/I_{eff}

The first analysis was conducted on the effective moment of inertia (I_{eff}) obtained from the finite element analysis results for all variations of castellated beam geometry. The calculated I_{eff} values were then correlated with the beam span length (L) to observe the trend of flexural stiffness

variation among the analyzed castellated beam models. Theoretically, as the ratio of span length to beam depth increases, the influence of shear deformation on the overall stiffness becomes smaller. Consequently, the value of I_{eff} tends to converge toward the true flexural stiffness of the beam.

The subsequent analysis compares the I_{nett} values obtained from analytical calculations with the I_{eff} values derived from numerical simulations. This comparison is used to validate the reliability of using I_{nett} in structural design practice and to determine whether a correction factor is required. Furthermore, the relationship between the span to depth ratio (L/d_g) and the stiffness ratio (I_{eff}/I_{nett}) is analyzed to determine the conditions under which the effect of shear deformation can be neglected. Also, to derive an empirical correction factor, if necessary, by using I_{nett} as an approximation of the effective moment of inertia in castellated beams.

To examine the influence of castellated beam geometry on its flexural stiffness, a correlation analysis was performed between the ratios $\beta = L/d_g$ and $n = I_{nett}/I_{eff}$, as illustrated in Figure 6. The data used to establish this correlation were derived from Table 3.

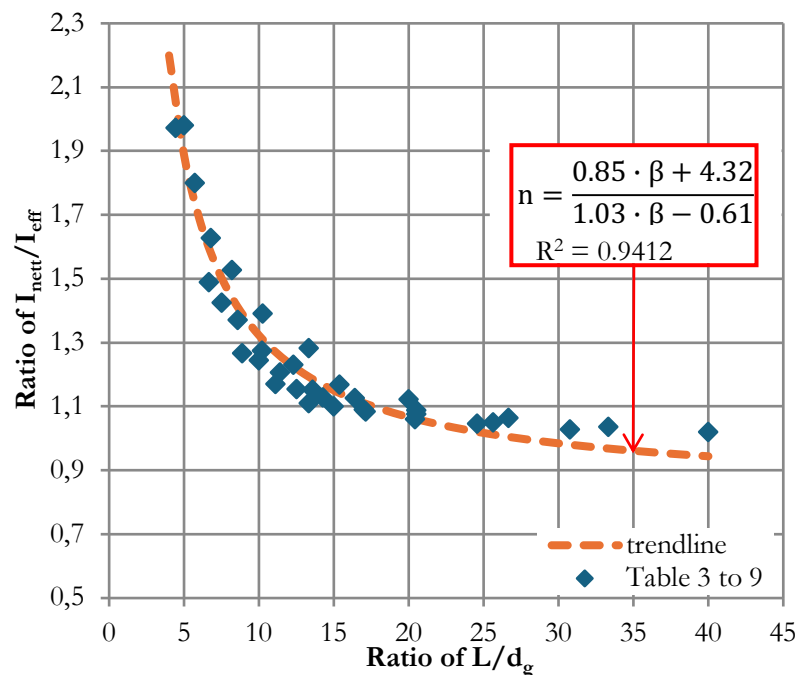


Figure 6. Correlation between span to depth ratio L/d_g and I_{nett}/I_{eff}

The graph of the analysis results shows that the value of I_{nett}/I_{eff} decreases and approaches convergence as the L/d_g ratio increases. At relatively small L/d_g values (less than 15), the stiffness ratio (I_{eff}/I_{nett}) is greater than 1.1, indicating that I_{eff} is still significantly affected by shear deformation. However, as L/d_g increases, the influence of shear deformation diminishes and the I_{nett}/I_{eff} ratio tends to approach unity. This indicates that, for certain L/d_g ratios, the net moment of inertia (I_{nett}) according to the AISC Design Guide 31 can be used directly as a practical design parameter without any correction factor.

Figure 6 shows the general trend of this relationship, where the I_{nett}/I_{eff} ratio becomes progressively stable and approaches a constant value as L/d_g increases. Based on regression analysis of the numerical data, the relationship between the two parameters can be expressed empirically with equation as follows:

$$n = \frac{0.85 \cdot \beta + 4.32}{1.03 \cdot \beta - 0.61}$$

Where:

n = ratio of I_{nett}/I_{eff}

B = ratio of L/d_g

This empirical relationship exhibits a strong fit with a coefficient of determination $R^2 = 0.9412$, indicating that approximately 94.12% of the variation in the $I_{\text{nett}}/I_{\text{eff}}$ ratio can be explained by changes in the L/d_g ratio.

This empirical relationship can serve as a practical guideline for evaluating whether the effect of shear deformation should be considered in structural design. Based on the results of this study, when $L/d_g \geq 18$, the stiffness ratio ($I_{\text{eff}}/I_{\text{nett}}$) is less than 1.1, meaning that I_{nett} can be considered equivalent to I_{eff} and can be safely used for castellated beam design in general engineering applications. For castellated beams with geometries within the scope of this study, when $L/d_g < 18$, the I_{nett} value obtained from the AISC Design Guide 31 can be corrected using the factor:

$$k = \frac{1}{n}$$

so that the effective moment of inertia can be estimated as:

$$I_{\text{eff}} = k \cdot I_{\text{nett}}$$

Therefore, the proposed empirical relationship provides a practical means for estimating the effective flexural stiffness of castellated beams while accounting for shear deformation effects, particularly for beams with relatively small L/d_g ratios.

Conclusion

This study compares the net moment of inertia (I_{nett}) according to the AISC Design Guide 31 with the effective moment of inertia (I_{eff}) obtained from Finite Element Analysis (FEA) for various configurations of castellated steel beams. The results indicate that the influence of shear deformation on the flexural stiffness of castellated beams decreases as the span to depth ratio (L/d_g) increases, causing the I_{eff} values to gradually converge toward the theoretical stiffness of the beam (I_{nett}). The comparison between I_{nett} and I_{eff} shows that the net moment of inertia generally provides conservative estimates and can still be used as a practical design reference within certain geometric limits. A strong correlation was found between the span to depth ratio $\beta = L/d_g$ and the stiffness ratio $n = I_{\text{nett}}/I_{\text{eff}}$, which can be expressed by an empirical relationship with a coefficient of determination $R^2 = 0.9412$. Based on the analysis results, when $L/d_g \geq 18$ the stiffness ratio ($I_{\text{eff}}/I_{\text{nett}}$) is less than 1.1, indicating that I_{nett} can be considered equivalent to I_{eff} and safely applied in structural design without additional correction, whereas for $L/d_g < 18$ the effect of shear deformation becomes more significant and the I_{nett} value can be corrected using the empirical factor $k=1/n$ to obtain a more accurate representation of the effective flexural stiffness of castellated beams.

Acknowledgment

The author would like to express sincere gratitude to Universitas Langlangbuana and the Institute for Research and Community Service (LPPM) of Universitas Langlangbuana for the financial support provided through the 2025/2026 academic year internal research grant, which enabled the successful completion of this study.

References

- Abhale, S. S., & Wakchaure, M. R. (2017). Study of Castellated Beam by Using Different Codes. *International Journal of Engineering Sciences & Research Technology*, 6(2), 748–751. https://www.ijesrt.com/Old_IJESRT/issues_pdf_file/Archive-2017/February-217/111.pdf

- Al-Thabthabee, H. W. A., & Al-Kannoon, M. A.-A. (2018). Improving Behavior of Castellated Beam by Adding Spacer Plat and Steel Rings. *Journal of University of Babylon for Engineering Sciences*, 26(4), 331–344. <https://doi.org/10.29196/jub.v26i4.810>
- Anbarasu, M., Pandey, A. K. P. K., Patton, M. L., & Carvalho, H. (2021). Testing and modelling of hot-rolled steel castellated hollow tubular beams. *Structures*, 34, 4025–4040. <https://doi.org/10.1016/j.istruc.2021.10.003>
- Barkiah, I., & Darmawan, A. R. (2021). Comparative analysis of the flexural capacity of conventional steel beams with Castellated beams. *IOP Conference Series: Earth and Environmental Science*, 780(1). <https://doi.org/10.1088/1755-1315/780/1/012013>
- Budi, L., Sukamta, & Partono, W. (2017). Optimization Analysis of Size and Distance of Hexagonal Hole in Castellated Steel Beams. *Procedia Engineering*, 171, 1092–1099. <https://doi.org/10.1016/j.proeng.2017.01.465>
- Deepha, R., & Jayalekshmi, S. (2020). Finite Element Analysis on Shear Strength of a Castellated Beam with Hexagonal Web Opening. *IOP Conference Series: Materials Science and Engineering*, 1006(1). <https://doi.org/10.1088/1757-899X/1006/1/012009>
- Deepha, R., Jayalekshmi, S., & Jagadeesan, K. (2020). Nonlinear analysis of castellated ISMB150 – I beam with hexagonal openings – A finite element approach. *Materials Today: Proceedings*, 27, A8–A16. <https://doi.org/10.1016/j.matpr.2020.09.369>
- El-Tobgy, H. H., Abu-Sena, A. B. B., & Fares, M. W. (2021). Experimental and parametric investigation of castellated steel beam-column in various expansion ratios, lengths and loading conditions. *Structures*, 33(December 2020), 484–507. <https://doi.org/10.1016/j.istruc.2021.04.053>
- Elaiwi, S., Kim, B., & Li, L.-Y. (2019a). Bending Analysis of Castellated Beams. *Athens Journal of Technology & Engineering*, 6(1), 1–16. <https://doi.org/10.30958/ajte.6-1-1>
- Elaiwi, S. S., Kim, B., & Li, L. (2019b). Linear and Nonlinear Buckling Analysis of Castellated Beams. *International Journal of Structural and Civil Engineering Research*, 8(2), 83–93. <https://doi.org/10.18178/ijscer.8.2.83-93>
- Elsa Sabu, D., & Joseph, D. (2022). a State-of-Art-of Review on Castellated Beam. *International Research Journal of Engineering and Technology*, 568–573. www.irjet.net
- Fares, S. S., Coulson, J., & Dinehart, D. W. (2016). AISC Design Guide 31: Castellated and Cellular Beam Design. American Institute of Steel Construction, 1–116.
- Frans, R., Parung, H., Sandy, D., & Tonapa, S. (2017). Numerical Modelling of Hexagonal Castellated Beam under Monotonic Loading. *Procedia Engineering*, 171, 781–788. <https://doi.org/10.1016/j.proeng.2017.01.449>
- Gunawan, D., & Suryoatmono, B. (2017). Numerical Study on Lateral-torsional Buckling of Honeycomb Beam. *Procedia Engineering*, 171, 140–146. <https://doi.org/10.1016/j.proeng.2017.01.320>
- H. Upadhyay, M., B. Patel, V., & A. Arekar, V. (2021). Parametric Study On Castellated Beam With Arch-Shape Openings. *International Journal of Civil Engineering*, 8(5), 52–57. <https://doi.org/10.14445/23488352/ijce-v8i5p106>
- Hadeed, S. M., & Alshimmeri, A. J. H. (2019). Comparative Study of Structural Behaviour for Rolled and Castellated Steel Beams with Different Strengthening Techniques. *Civil Engineering Journal*, 5(6), 1384–1394. <https://www.civilejournal.org/index.php/cej/article/view/1532/pdf>
- Haris, S., Sari, P. K., & Masrilayanti. (2023). Numerical Study of Castellated Beam with Stiffener -

- Case on Cantilever Structure. IOP Conference Series: Earth and Environmental Science, 1173(1). <https://doi.org/10.1088/1755-1315/1173/1/012012>
- Hoseinpour, H., Valluzzi, M. R., Garbin, E., & Panizza, M. (2018). Analytical investigation of timber beams strengthened with composite materials. *Construction and Building Materials*, 191, 1242–1251. <https://doi.org/10.1016/j.conbuildmat.2018.10.014>
- Huang, B., & Cao, S. (2025). Lateral-torsional buckling performance of I-section cellular beams. *Journal of Constructional Steel Research*, 228. <https://doi.org/10.1016/j.jcsr.2025.109417>
- Kim, B., Li, L.-Y., & Edmonds, A. (2016). Analytical Solutions of Lateral–Torsional Buckling of Castellated Beams. *International Journal of Structural Stability and Dynamics*, 16(8), 1–16. <https://doi.org/10.1142/S0219455415500443>
- Kowsalya, M., & Iyappan, G. R. (2020). Study on Castellated Web Beam with Optimized Web Opening – State of the Art Review. *International Journal of Research Publication and Reviews*, 1(7), 87–93.
- Kumar, R., Structural, M. E., & Jagadeesan, K. (2015). Experimental Study on Castellated Beam to Enhance the Shear Strength. *International Journal of Engineering Research & Technology (IJERT)*, 3(16), 3–6. <https://www.ijert.org/experimental-study-on-castellated-beam-to-enhance-the-shear-strength>
- Kwani, S., & Wijaya, P. K. (2017). Lateral Torsional Buckling of Castellated Beams Analyzed Using The Collapse Analysis. *Procedia Engineering*, 171, 813–820. <https://doi.org/10.1016/j.proeng.2017.01.370>
- Liu, M., Liang, M., Ma, Q., Wang, P., & Ma, C. (2020). Web-post buckling of bolted castellated steel beam with octagonal web openings. *Journal of Constructional Steel Research*, 124(105794). <https://doi.org/10.1016/j.jcsr.2019.105794>
- Maali, M., & Cinar, N. (2024). Experimental and Numerical Tests on Beams with Web Openings Under Cyclic Loading. *International Journal of Civil Engineering*, 23, 149–167. <https://doi.org/10.1007/s40999-024-01025-5>
- Mehetre, A. J., Talikoti, R. S., & Sonawane, P. B. (2020). Experimental Research on Equivalent Rectangular Opening Castellated Beam with Fillet Corner. *International Journal of Recent Technology and Engineering (IJRTE)*, 8(5), 5415–5420. <https://doi.org/10.35940/ijrte.E7103.018520>
- Morkhade, S. G., & Gupta, L. M. (2017). Experimental investigation for failure analysis of steel beams with web openings. *Steel and Composite Structures*, 23(6), 647–656. <https://doi.org/10.12989/scs.2017.23.6.647>
- Najafi, M., & Wang, Y. C. (2017). Behaviour and design of steel members with web openings under combined bending, shear and compression. *Journal of Constructional Steel Research*, 128, 579–600. <https://doi.org/10.1016/j.jcsr.2016.09.011>
- Qiao, H., Guo, Z., & Chen, Y. (2022). Experimental investigation of a substructure in a frame with castellated steel beams in case of a column loss. *Engineering Structures*, 255(113926). <https://doi.org/10.1016/j.engstruct.2022.113926>
- Sandeep, R. C., Kumar, K. M., Madhavarao, G., Thirumalairaja, R., & Manikandan, C. (2020). Behavior of Castellated Beams with and Without Stiffeners. *International Journal of Innovative Research in Computer Science & Technology (IJIRCST)*, 8(5), 394–399. <https://doi.org/10.55524/ijircst.2020.8.5.12>
- Serene, K. T., & Aswathy, P. (2019). Finite Element Analysis of Composite Beams and Columns with Castellated Members. *International Journal of Scientific & Engineering Research*, 10(5).

- https://www.thejusengg.com/ckfinder/userfiles/files/7_Finite-Element-Analysis-of-Composite-Beams-and-Columns-with-Castellated-Members.pdf
- Shaikh, A. S., & Autade, P. B. (2016). Structural Analysis and Design of Castellated Beam in Fixed Action. *International Journal of Innovative Research in Advanced Engineering (IJIRAE)*, 3(08), 92–97. <https://www.irjet.net/archives/V3/i8/IRJET-V3I834.pdf>
- Shiyekar, S. M., Gawade, S. S., Rathore, H. K., Hendre, R. D., & Solanke, V. M. (2024). Load Behaviour of Steel Beam with Opening using Finite Element Approach. *International Journal for Research in Applied Science & Engineering Technology (IJRASET)*, 12(V), 4832–4840. <https://doi.org/10.22214/ijraset.2024.62766>
- Sonck, D., & Belis, J. (2016). Weak-axis flexural buckling of cellular and castellated columns. *Journal of Constructional Steel Research*, 124, 91–100. <https://doi.org/10.1016/j.jcsr.2016.05.002>
- Sonck, D., & Belis, J. (2017). Lateral-Torsional Buckling Resistance of Castellated Beams. *Journal of Structural Engineering*, 143(3), 1–9. [https://doi.org/10.1061/\(asce\)st.1943-541x.0001690](https://doi.org/10.1061/(asce)st.1943-541x.0001690)
- Subramani, T., & Sukumar, V. (2018). Castellated beam with and without stiffeners using ANSYS. *International Journal of Engineering and Technology(UAE)*, 7(3), 94–97. <https://doi.org/10.14419/ijet.v7i3.10.15638>
- Wakchaure, M. R., Sagade, A. V., & Auti, V. A. (2012). Parametric study of castellated beam with varying depth of web opening. *International Journal of Scientific and Research Publications*, 2(8), 2250–3153. www.ijsrp.org
- Wang, C. K. (1983). *Intermediate structural analysis*. McGraw-Hill Book Company.
- Wang, P., Guo, K., Liu, M., & Zhang, L. (2016). Shear buckling strengths of web-posts in a castellated steel beam with hexagonal web openings. *Journal of Constructional Steel Research*, 121, 173–184. <https://doi.org/10.1016/j.jcsr.2016.02.012>
- Weidlich, C. M., Sotelino, E. D., & Cardoso, D. C. T. (2021). An application of the direct strength method to the design of castellated beams subject to flexure. *Engineering Structures*, 243(112646). <https://doi.org/10.1016/j.engstruct.2021.112646>
- Yuan, W. bin, Yu, N. ting, Bao, Z. shui, & Wu, L. ping. (2016). Deflection of castellated beams subjected to uniformly distributed transverse loading. *International Journal of Steel Structures*, 16(3), 813–821. <https://doi.org/10.1007/s13296-015-0120-2>
- Yustisia, V. W., Suswanto, B., Irawan, D., & Iranata, D. (2020). The structural behavior of castellated beam with shape variation using finite element methods. *IOP Conference Series: Materials Science and Engineering*, 930(1). <https://doi.org/10.1088/1757-899X/930/1/012051>
- Zarmihan, R. P., Pratiwi, V., & Krisnadi, S. (2023a). Effect of Cross-sectional Height on IWF Profile Strength, Hexagonal Beam, and Octagonal Beam. *The 1st International Student Conference on Engineering and Environmental Research (ISCEER2022)*, Volume 2882, Issue 1. <https://doi.org/10.1063/5.0175643>
- Zarmihan, R. P., Pratiwi, V., & Krisnadi, S. (2023b). Studi Perbandingan Kekuatan Profil IWF, Hexagonal Beam dan Octagonal Beam dengan Perhitungan Manual dan Metode Elemen Hingga. *Cantilever: Jurnal Penelitian Dan Kajian Bidang Teknik Sipil*, 11(2), 129–140. <https://doi.org/10.35139/cantilever.v11i2.165>
- Zhou, X., Li, J., He, Y., He, Z., & Li, Z. (2018). Finite element analysis of thermal residual stresses in castellated beams. *Journal of Constructional Steel Research*, 148, 741–755.

<https://doi.org/10.1016/j.jcsr.2018.06.026>