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Design of Realistic Mathematics Education Approach to Develop Students High-Order Thinking Skill in Online Learning

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Abstract

High-level mathematical thinking skills are the main goal in learning mathematics. So far, the practice of learning in schools has not facilitated students to develop these abilities. PMRI (the Indonesian version of realistic mathematics education) is a learning approach that is oriented towards the students` ability to understand the realistic situations that can be imagined and integrated with the ability to understand concepts, use progressive mathematical models, interactivity and take advantage of student contributions are expected to improve students' abilities in learning mathematics. This study aims to determine the implementation of PMRI approach in improving high-order thinking skills of mathematics in class XI SMA Mutiara Bangsa 2 Tangerang on limit function through e-learning. This research was developed based on a research design that includes three, those are preliminary design, experiment, and retrospective analysis. This study was designed with five meetings using six research subjects. The subjects of this study consisted of six students who were selected based on their evaluation test and their activeness during learning process. The research instruments analyzed were formative and summative test results based on higher-order thinking skills, findings note, student questionnaires, and the hypothetical learning trajectory (HLT). The results obtained from the retrospective analysis show that the implementation of PMRI in learning and instruction of limit function is able to develop higher-order thinking skills of class XI students of SMA Mutiara Bangsa 2 Tangerang.

Key words: design research, high-order thinking, PMRI, limit, e-learning

INTRODUCTION

The global community is required to think creatively to bring up ideas and create notion that may not have been thought of before. Creative thinking requires abilities to find problems around, analyze and find the best solution method, by making something new or adding value to a certain product so that someone will become a problem solver needed by society (Judkins, 2015). The challenge of global society is the ability to think dynamically adjusting to changing times, solving problems, making new innovations and creative breakthroughs (Hallissy, 2012). Therefore, the school education curriculum, one of which is learning mathematics, must be arranged to be able to train students' thinking processes. Problem is a situation where someone face a problem and don't know how to solve it, so that creativity, problem-solving and high-order thinking are required (Seifert, K & Sutton, 2011).

According to Pellegrino, J.W & Hilton (2012) an important asset for the success of students in this globalization era is education that focuses on developing cognitive abilities (IQ) and non-cognitive skills as well as psychological development. In accordance with the National Educational System law number 20 of 2003, education is a conscious and planned effort to create a learning atmosphere and learning process so that students actively develop their potential to have religious spiritual strength, self-control, personality, intelligence, noble character, and good skills needed by himself, society,

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nation and state. Referring to these provisions, the self-potential needed by students to be able to live as role of humanist is the integration of cognitive, affective and psychomotor abilities (Sidiknas, 2003).

Mathematics High-Order Thinking Skill

The implementation of learning, especially teaching mathematics in schools, the process of creative thinking in solving problems is included in high-order thinking skills. According to (Gershon, 2015) places the ability of creative thinking (Bloom's Taxonomy) processes in the top three abilities that students must master in mathematics learning which includes analysis, synthesis, and evaluation. This case can help students develop reasoning idea, argue well, be able to solve problems and construct explanations, also hypothesize and understand complex problems clearly (Dinni, 2018). Based on Bloom's Taxonomy, thinking skills are grouped into two parts, the first is low-order thinking skills which include the ability to remember, understand, and apply and the second part is the ability to think at a high level (high order thinking) which consists of the ability to analyze, evaluate, and create. Furthermore Anderson, L. W. & Krathwohl (2006) made the description and revision keywords of Bloom's Taxonomy presented in table below:

TABLE 1. Revised Bloom's Taxonomy

THINKING LEVEL	CATEGORY	KEY WORD
HOTS	*Creating*: can students create an object or view?	• Create own ideas. • **Action Verb**: construct, design, create, develop, write, formulate.
	Evaluating: can students declare good or bad about a particular phenomenon or object?	• Make own decisions. • **Action Verb**: evaluate, judge, argue, decide.
	Analyzing: can students sort the parts based on their differences and similarities?	• Specifies aspects / elements. • **Action Verb**: compare, examine, criticize, test.

Higher-order thinking skills in the learning process should facilitate students' mathematical literacy processes to formulate, apply and interpret mathematics in various contexts, including the ability to do mathematical reasoning and use concepts, procedures, and facts to describe, explain or predict phenomena or events (OECD, 2013).

Learning that develops higher order thinking skills is different from conventional learning in general, which is oriented towards improving the quality of thinking processes during learning. According to Limbach, Barbara & Waugh (2006) developing high-order thinking processes can be solved with neatly structured learning designs and enriching teacher creativity in carrying out learning. Furthermore Brookhart (2010) there are three basic principles in conducting assessments that develop students' higher order thinking skills, namely; (1) *Use introductory materials or allow access to resources material*, asking students to use information sources as material for thinking, that's mean in assessing higher order thinking skills the teacher must be able to make questions that can stimulate students' thinking processes. (2) *Use novel material*, means asking students to really think, not just remembering the material that has been learned during the learning process in the classroom by integrating various knowledge they have to solve problems. (3) *Attend separately to cognitive complexity and difficulties*, organize the difficulty level and cognitive complexity level questions separately. The teacher must be able to distinguish between the level of difficulty and the level of complexity (low-level thinking or high-level thinking).

Limit is one of the compulsory materials for students to learn calculus at an advanced level. Through of the Liang (2016) research teaches the concept of limit functions using the Conceptual Conflict Strategy and Desmos Graphing Calculator, argues that many students experience failure in limit material due to their lack of basic conceptual skills. Developing students' mathematical thinking skills, there are two main objectives of the teacher in teaching, namely teaching content, facilitating

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conditions that support learning and instilling an understanding that mathematics is very useful and close to everyday of life.

Indonesian Version of Realistic Mathematics Education (PMRI)

The Indonesian version of realistic mathematics education (PMRI) is the result of an alignment of the mathematics learning method from the Netherlands, that developed by the Freudenthal Institute. According to Stemn (2017) RME is used to facilitate students develop their mathematical thinking concepts by emphasizing the use of contextual learning conditions and visual representations in learning. Activities in the realistic mathematics learning process are arranged and prepared based on real problems such as fictional stories, games or even formal forms of mathematics as realistic problems that are useful for building mathematical concepts or as a source for learning (Soviawati, 2011). Meanwhile, Heuvel (2003) states that developing realistic mathematics education is used as a learning model to improve the quality of mathematics learning, through an observation activity. The main essence in the process of a realistic learning approach is try to bring mathematics learning closer to life around students, because in fact mathematics is born from phenomena in the natural environment which are generalized by scientists so that can be learned by everyone. According to (Sembiring, 2010) other essences of realistic mathematics education in Indonesia are three key principles that can be used as the basis for designing learning, those are guided rediscovery, didactic phenomenology, and the principle of self-building models. furthermore Suherman (2003) argue that characteristics possessed by Indonesian version of realistic mathematics education are using real-world contexts, models, student production and construction, interactivity, and intertwinement.

E-Learning Instruction

The rapid development of technology in modern times will automatically affect the school learning process. Educators and students can take advantage of the role of technology to facilitate the learning and teaching process that is monotonous and tends to be teacher-centered. There are many innovations, facilities, and variations of technology-based learning that can be integrated with the concept of learning in the classroom in order to stimulate students' interest in following lessons. One of the uses of technology in this modern era is electronic learning or more commonly known as E-Learning. Through this method, it will be able to reduce the interaction between students and teachers even though they are not in the same place without reducing the quality of learning. According to Bates (2015) revealed that there are three important components in online instruction to be able to create a community of inquiry, namely: (1) *Social presence*, which is an essential aspect so that students pro-active in the learning process through participation. (2) *Teaching presence*, plays a role in designing, facilitating, and directing student knowledge and supervising social processes in the form of learning discussions. (3) *Cognitive presence*, is something that must be prepared and presented so that students achieve their goals and gain knowledge during the learning process. Furthermore, Hamidy, Anwaril & Purboningsih (2015) provide an opinion that collaborative online instruction is a learning that focuses on the success of process through a series of activities such as proposing ideas, asking criticism, questions, or input as well as providing reflection on an idea. in practice e-learning uses two approaches that are commonly used, namely, self-paced e-learning when students learn independently through materials prepared by the teacher and instructor-led and facilitated e-learning where the teacher interact directly with students to achieve predetermined goals (Girardini, 2017). Based on this case, it is necessary to explore deeply how to develop instruction with RME approach through e-learning which can provide benefits in increasing high-level mathematical thinking skills as practically being able to be academic studies and further research in order to develop effective learning.

METHOD

The type of research that will be used is development research in the form of design research which will be carried out on six participants as research subject in the even semester 2019/2020 which

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selected based on evaluation test score and activeness during the learning process. The subject chosen is the Limit Function of class XI SMA Mutiara Bangsa 2 Tangerang to improve high-level of mathematical thinking skills with the Indonesian version of realistic mathematics education approach. Activities and problems given during the research process are context problems that encourage students to develop mathematic higher-order thinking skills. Formative and summative tests data based on high-order thinking skills, record findings, student questionnaires and cross-learning hypotheses as information were interpreted during the retrospective analysis process. According to Gravemeijer (2006) the implementation stages of design research are (1) preliminary design; in this phase, a Hypothetical Learning Trajectory (HLT) or Cross Learning Hypothesis is made. In this case, HLT is made to anticipate events about what might happen to students during the learning process of mathematics, both students' thinking processes and things that will happen during the learning process. (2) experiment; in this phase, the designs that have been designed are tested in the field. This stage allows the adjustment of whether the hypotheses that have been predicted at the preliminary design stage occur in accordance with the predictions or not. (3) retrospective analysis; the data obtained during the previous study were analyzed based on the HLT which was made with the activities that actually occurred during the learning process.

Data Analysis

The interpretation framework of this study is based on the relationship between social perspectives (class community) and psychological perspectives (individual students) which are analyzed simultaneously on three things. The first one, covering the development of individual students' mathematical thinking processes and how the role of student thinking contribution, the second one, role of teachers and classroom culture, and the third one, possible improvements to the instructional design. Here are the results of teaching experiment:

a. First Concourse: Develop Students' Understanding of the Definition and Concept of Limits Through Context Problems.

   The activity at the first concourse was to find the definition of limit intuitively through context problems and to understand the concepts of left and right hand limits. The activity begins with a question and answer session discussing the meaning of limit based on students' understanding in day life through instructor-led e-learning. That limit is a boundaries or approaching a situation. The context used is an illustration of a soccer match, real numbers that converge to 5 and the value of phi based on the Archimedes experiment.

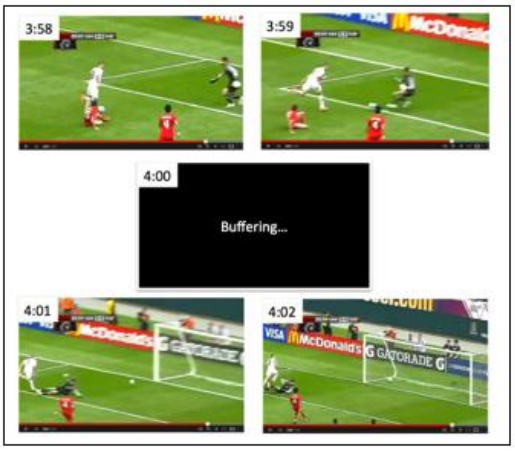

FIGURE 1. Illustration of Soccer Match

Teacher : *what do you think about limit?*
A1 : *boundary that is closed to a number*
A3 : *restriction of a thing*
Teacher : *please give examples about limit in day life*
SP 6 : *fill the glass with water*

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A4 : *limit of a credit card*
Teacher : *why the example you give is limit?*
SP 6 : *yah, so if we fill water in the glass and so as not to spill, just fill the glass be close to full*
Teacher : *how to know the ball position when buffering?*
SP 2 : *rewind slowly*
A2 : *by replay the video*
SP 1 : *by repeating or rewinding the video*

The first activity was continued by discussing real numbers that approach to five from the either left and right hand also determining the value of phi based on Archimedes' experiment.

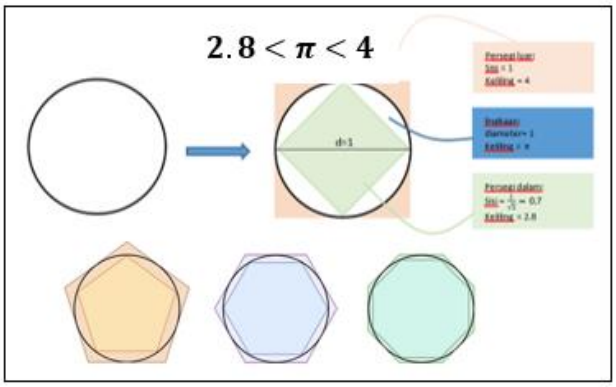

FIGURE 2. Illustration Archimedes Experiment

Teacher : *please mention the real numbers that converge to 5*
SP 2 : *4,99999999*
A4 : *1- 4,999*
Teacher : *then give an opinion whether 5,0005 is a real number which converge to*
SP 1 : *yes, but a bit greater than*
Teacher : *can you give another examples related to real number which converge to 5?*
SP 1 : *…4,55; 4,75; 4,99; 4,999…. (before 5) 5,01; 5,005; 5,05; 5,06… (after 5)*
Teacher : *based on Archimedes' experiment in determining the value of phi what happens if there are more sides of polygon inside and outside of?*
SP 5 : *if there are more sides will get closer to the original shape*
Teacher : *what shape?*
SP 5 : *circle*
A1 : *it is easier to find the circumference of*
A3 : *if there are more side will be getting like a circle*

Based on the HLT predictions made, that is students are able to understand the meaning of limit. the discussion activities at the beginning of the concourse are able to develop students' understanding of the meaning of limit based on the given context problem activity. There were students who gave inappropriate opinions about the concept of the meaning of limit, but with teacher's guided the students were able to understand the concept correctly. Through these contributions, students are able to develop an understanding of the meaning of limit, that limit is a boundaries or approaching a value from either the left or right hand. It can also be seen that students are able to provide examples of limits in day life based on own student's understanding.

The next activity, students were given first worksheet with context problems as a local government plan illustration that would make a horse racing field, with the prior knowledge of limits concept then tried to determine the length of the track using the Archimedes experiment.

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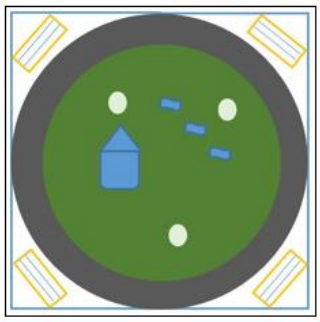

FIGURE 3. Horse Racing Field Illustration

Based on the overall work results, it can be seen that students are able to identify and find ideas on problems, students are also able to integrate various concepts with other materials in developing resolution strategies. At the end of the question students must make generalizations by analyzing the pattern of relationships formed by the left and right hand limits. There are students who are inaccurate in giving conclusions due to the inability to connect the concept of limit by approaching a value of the answers that have been made. Overall activities at the first meeting based on RME were able to develop high-order mathematical thinking skills.

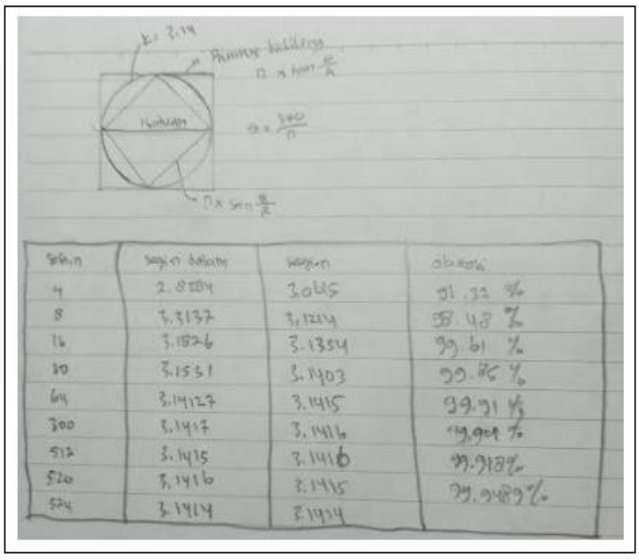

FIGURE 4. Student answer of first worksheet

b. Second Concourse: develop students' understanding of the limit theorem of algebraic functions.

Activities at the second concourse begin with apperception regarding to the limit concept that has been studied in the previous activity and discuss the graphical illustration of the function at plane diagram when the function with x approaches c from the left and right hand and $x \neq c$, then the value of the function $f(x)$ is close to L.

Teacher : *what do you think about function?*
SP 1 : *a function is a formula for processing input into output with a particular rule*
SP 4 : *function is an equation that maps the set of element x to the set of element y*
Teacher : *based on the illustration of the function in plane diagram, which were belongs to the left hand limit?*
A1 : *before L*
SP 2 : *1, 2, 3…. before L*
SP 1 : $l_1, l_2, l_3, l_4, \ldots L$
Teacher : *what if the left and right hand limit are not approaching same certain value?*
A3 : *there is no limit*
SP 2 : *that's mean not a limit function*

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Based on the HLT predictions made, that is students are able to understand the concepts and standard limit theorems. Through these activities students have no difficulty recalling the concept of domain, range and value of a function and integrating the concept of algebraic limit function to determine the limit value if the left and right hand limits are both close to a certain value. Activities at the second concourse began with apperception regarding the limit concept that had been learned in the previous meeting. The context presented at this concourse is the movement of a bee flying in a parabolic trajectory from 1-2 seconds, then when it reaches the maximum altitude it flies constantly at a distance of 5 meters from the ground for an interval of 2-3 seconds, at the last second the bees fly vertical dive down to the ground.

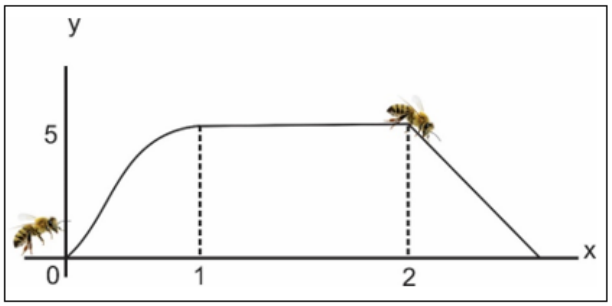

FIGURE 5. Illustration of Bee Fly

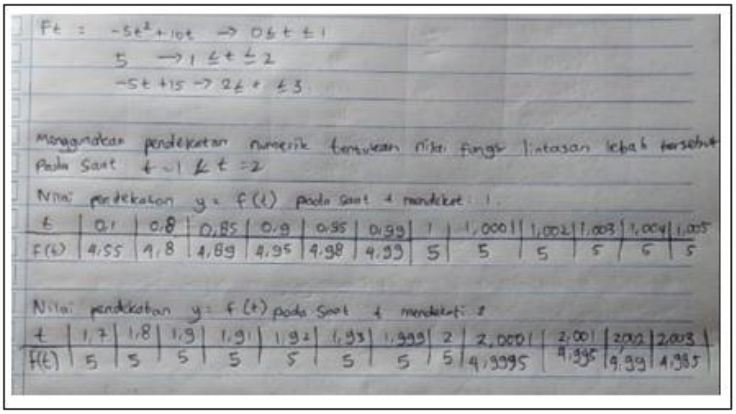

FIGURE 6. Student answer of second worksheet

Base on the figure 6. show that students were able to identify problems and develop solution strategies through integrating various concepts such as straight line motion, acceleration, linear equations through two-point form. Students were also able to test values when approaching t = 1 and t = 2 using the numerical table and used in making generalization based on the limit theorem. student work activities at the second concourse are able to develop students' high-order mathematical thinking skills based on the success of completing and answering questions on the second worksheet problems given. The teacher then emphasizes the concept of the limit theorem on algebraic functions that students have learned from the previous term and provides reinforcement of the explanation that to determine value of an algebraic function is to analyze the boundaries pattern from the left and right hand limit, if the both approaches to the same value, it can be said that the function has a value of limit.

c. Third Concourse: develop students' understanding kind of limit functions.

Activities at the third concourse have used problems by presenting a formal mathematical form which is analyzing function graphs and providing appropriate arguments based on the concept of limits. Previously, students and teachers in the discussion forum addressed the undefined, zero, and infinite forms through the context of distributing pencils to the children.

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FIGURE 7. Illustration of Children Pencil Distribution

Teacher : *what do you think about undefined?*
SP 6 : *the undefined means the value is impossible to gain*
A1 : *no or without explanation*
SP 4 : *if divided by zero*
Teacher : *please give another example in day life about zero*
SP 6 : *if expect to eat there is not exist of food*
A1 : *if want to make summary but no stationary*
Teacher : *what do you think about infinite?*
SP 6 : *endless*
SP 1 : *a value that converge to c (constant real number), but never touch that value*
Teacher : *by so the infinite limit when there is a fraction whose denominator is close to zero then the value will be positive infinite or negative infinite*

In accordance with the HLT prediction that has been made previously, that is students can understand undefined, zero and infinite limit function. the results are through discussion forum activities students can understand the concept kind of limits. students can understand that if there

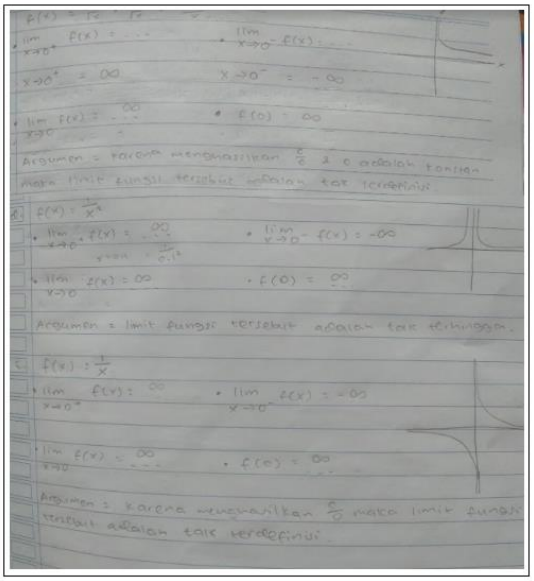

FIGURE 8. Student answer of third worksheet

are six pencils, while no children will be divided, there is no result in other words undefined. If there are no pencils to be distributed, while there are three children, the result is that none of the children will get pencils or the children will get zero pencils. The activity was continued by giving

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the student worksheet to analyze a limit in the form of a fraction and instructing them to define the kind of limit by calculating with limit theorem and drawing a graph of the function that has an asymptote, so that the value of the limit function can be clearly identified.

Most of the students were able to solve the problems given, there were only a few students who gave incorrect answers because they could not distinguish between undefined and infinite forms. At the end of the concourse the teacher gave students an understanding that if a fraction limit result $\frac{c}{0}$ and x approaching to a nonzero then the limit is undefined, while if a fraction limit result $\frac{c}{0}$ and x approaching to 0 then the limit is infinite.

d. Fourth Concourse: develop students' understanding in determining the existence of a limit function.

Activity at the fourth concourse begins by paying attention to the limit function illustration of $\lim_{x\to 0} \frac{f(x)}{g(x)}$ by the limit function theorem, it can be converted into $\frac{\lim_{x\to 0} f(x)}{\lim_{x\to 0} g(x)}$ and with the numeric graphs, it is known that if $f\ (x)$ and $g\ (x)$ both approach to zero it indicates that the direction of the value are different, so it's so-call the fraction function is of indefinite yielding $\frac{0}{0}$.

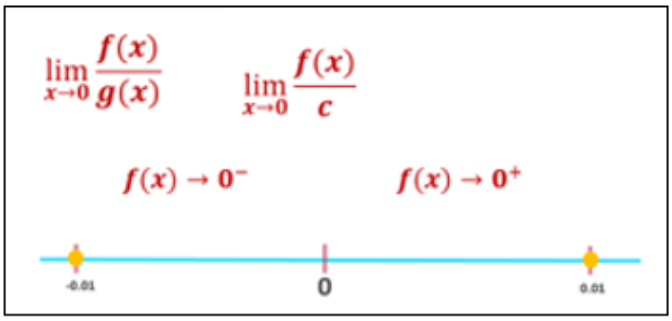

FIGURE 9. Illustration of Undefined Limit Function

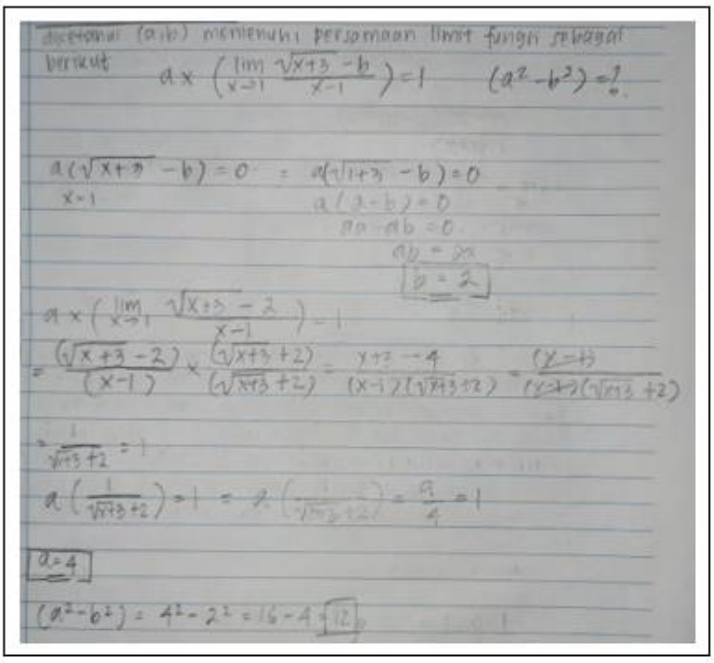

FIGURE 10. Student answer of forth worksheet

Through discussion activities assisted by the teacher as a facilitator by directing their understanding. Based on the HLT that has been designed, that is students can understand concept of indeterminate limit function and show the existence of. the results are students are able to develop the concept of an undefined limit and show that the limit function has a value, the process that must be done is manipulating by factoring or rationalizing the fraction until function $f\ (x)$ and $g\ (x)$ are

obtained so that the limit value can be determined. After students are able to understand the concept of undefined limit, the activity is continued by giving forth worksheet. The problems presented are already in the form of formal mathematics by combining various more complex concepts, the aim is that students can develop their ability to solve problems that require high-order mathematical thinking skills, based on the ability to model problems in the form of contextual presented at previous terms. Base on image 9 students have no difficulty in finding ideas, analyzing problems, and evaluating the answers they got. There were some students who experienced technical errors in answering the questions on worksheet four due to inaccuracy in writing number marks in substitution process and not checking the answers made so that the mistakes in the initial steps bring over the final result was incorrect. The teacher emphasizes students to be more careful and thorough in understanding and solving problems so that no mistakes occur at the end of learning.

CONCLUSION

According to the results of retrospective analysis and data analysis through the developed of hypothetical learning trajectory (HLT), it can be concluded that the design of learning activities based on the characteristics of Indonesian version of realistic mathematics education (PRMI) through e-learning instruction model can improve students' high-order mathematical thinking skills. This can be seen from the development of students' conceptual understanding to the limit function material and problem-solving abilities given to students in each learning activity. The mathematical context used on each worksheet is able to bridge students' thinking levels from the *model of* in the form of realistic situations such as illustrations of soccer games, horse racing fields and the pencils distribution to the children into *model for* in the form of more complex mathematical problems. The interactivity created during the learning process is carried out well in accordance with the previously designed of HLT, besides the creativity of students in expressing opinions or arguments and problem solving processes as a student contribution to knowledge expansion by integrating prior knowledge of mathematical concept or other material concept.

DISCUSSION

Based on the findings during the study, a better learning trajectory design is needed, especially when implementing e-learning instruction which provides access to fellow students and teachers face to face online. In addition to sufficient time adjustments, it is necessary to explore deeply about the use of contexts that are closer to life so that students can more easily understand the material to be delivered. In order to carry out further research, it is necessary to design an appropriate media such as a video application so that the process and task of the teacher in guiding student understanding can run well. It is necessary to choose a context and model that is closer to the daily lives, and to optimize the interaction of students' contributions to play an active role during learning. The hope is that with the refined learning trajectory design, the learning and instruction activity can be more effective and efficient, furthermore, the goal in developing understanding can occur more optimally. It is also necessary to design a learning trajectory that adjusts the learning topic with time allocation planned.

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