E-ISSN: 2621-4296

The Use of ICT and the Realistic Mathematics Education on Learning Functions to Develop Algebraic Thinking Skills

Mimi Nur Hajizah[1, a)], Dwi Antari Wijayanti[2, b)]

[1,2] *Universitas Negeri Jakarta*

Emails: [a)] miminurh@unj.ac.id, [b)] dwi-antari@unj.ac.id

Abstract

The concept of function has evolved into one of the most fundamental principles in modern mathematics, influencing practically every area. However, it proved to be one of the most difficult ideas to grasp in mathematics. The concept of function is frequently taught without any connection to real-life situations. Students must work with problem scenarios that happen in a variety of contexts that are relevant to their everyday lives or make sense to them in order to gain authentic comprehension. This is consistent with the RME's properties. The unwillingness to highlight the visual element of the notion is one of the major roadblocks in learning functions. Using ICT and an RME approach to learning functions as an alternative to overcome these barriers could be a viable option. ICT and RME can help students learn mathematical principles in function content by providing a clearer visual experience. The purpose of this study is to investigate how ICT and RME can be used to improve learning function and increase algebraic thinking skills. This study conducted in a classroom of 29 junior high school students at the eighth grade. Design research methodology was used in this study by conducting three phases: preliminary design, teaching experiment, and retrospective analysis.

INTRODUCTION

Algebra is a discipline of mathematics concerned with the formation of quantities, relationships, and structures. Use symbols to symbolize something unclear, such as variables, parameters, or something unknown, in order to learn these things. Algebra deals with resolving equations, analyzing functional relationships, and determining the structure of a representative system made up of expressions and relationships using symbols and extended numbers. Algebra is a tool for modeling real-world occurrences and addressing issues connected to various situations, not for resolving equations, assessing functional relationships, or establishing the structure of a representational system (Kieran, 1989; Lew, 2004; Radford 2006).

In general, students continue to struggle with mathematics (Jupri & Drijver, 2016; Bora & Ahmed, 2019; Feretti, 2019). Low algebraic thinking skills cause students to struggle with solving algebraic problems, simplifying algebraic equations and expressions, and interpreting graphs of functions. Due to these challenges, students learn by memorization rather than understanding the subject. As a result, children are only able to answer routine issues at a low level. Thinking algebra, according to Kieran, can be defined as a method of approaching quantitative situations that emphasizes aspects of general relations using tools that do not necessarily require the use of a letter symbol, but can eventually be used as a cognitive support to introduce and retain more traditional school algebra material (Kieran, 1989).

Students frequently encounter circumstances involving functional relationships in their daily lives. As a result, they can bring a lot of useful information into the classroom. This understanding can aid students in solving algebraic issues. Students' mathematical understanding, on the other hand, will not

E-ISSN: 2621-4296

promote new learning if it is not linked to formal algebra study (Kalchman & Koedinger, 2005). The concept of function has evolved into one of the most fundamental principles in modern mathematics, influencing practically every area. However, it proved to be one of the most difficult ideas to grasp in high school mathematics (Eisenberg, 2002; Michelsen, 2006). The concept of function is frequently taught without any connection to real-life situations. Students are occasionally introduced to formal mathematical representation of ideas like as f(x) too early, which can lead to misconceptions regarding functions. Students may struggle to understand the notion of function when it is given in a mathematical context that they do not understand.

Contextual and conceptual approaches are covered by a number of frameworks. The emergent model notion (Gravemeijer, 1999) has its beginning point in a specific problem circumstance that is then modeled. Giving students a problem/context at the start of their study allows them to create specialized situation approaches and symbolization. Furthermore, procedures and symbols are mathematically modeled, and models develop from student activities in this way. The model begins as a situation, evolves into an entity, and then is used as a model for mathematical reasoning. The transition from 'model of' to 'model for' should coincide with changes in how students see and think about models, moving away from models generated from the context of the situation and toward models produced from mathematical content. Students must work with problem scenarios that happen in a variety of contexts that are relevant to their everyday lives or make sense to them in order to gain authentic comprehension. This is consistent with the RME's properties.

The unwillingness to highlight the visual element of the notion is one of the major roadblocks in learning functions (Eisenberg, 2002). Students have a tendency to think of function solely in terms of symbolic representation. Visually, functions and related ideas are not grasped. This non-visual approach to learning about function stymies a person's ability to comprehend function. Using ICT and an RME approach to learning functions as an alternative to overcome these barriers could be a viable option. ICT and RME can help students learn mathematical principles in function content by providing a clearer visual experience. The purpose of this study is to investigate how ICT and RME can be used to improve learning function and increase algebraic thinking skills.

METHOD

The design research methodology was used in this study. There were three reasons for choosing this methodology: first, it provided useful perspectives for developing theory. Second, the outcomes were specifically valuable. Third, it made the researcher personally involved in the improvement of mathematical instruction. A design research study is divided into three phases: the preliminary design, the teaching experiment, and the retrospective analysis.

The preliminary design phase begins with identifying the learning obstacle, clarifying the objectives of learning mathematics, combined with anticipating teaching experiments by imagining how the teaching and learning process can be realized in the classroom. This first step produces an explicit formulation of a conjectured local instruction theory which consists of three components: a. learning objectives for students, b. instructional activities that are planned and the learning media that will be used, and c. the alleged learning process that anticipates how students' thinking and understanding can develop when instructional activities are applied in the classroom. This conjectured local instruction theory is open to adaptation based on input from students and evaluation of students' real understanding. This theory also reflects the importance of anticipating possible learning processes that can occur when instructional activities are applied in the classroom.

In the teaching experiment phase, the initial design is implemented. The process of anticipation and testing is basically a repetition process that resembles the idea of a mathematics learning cycle proposed by Simon (1995). Classroom learning activities allow researchers to investigate whether students' mental activity is as anticipated. Insights gained from a teaching experiment can be the basis for designing or modifying a sub-sequence of instructional activities and for constructing new assumptions about the expected mental activity of students. In this way, instructional activities are piloted, revised, and designed every day or every meeting during the teaching experiment.

The retrospective analysis aimed to reconstruct an improved local instruction theory based on what was learned from a series of micro-design cycles in a teaching experiment. The actual data analysis begins during the process of the teaching experiment being applied and will be complemented by a retrospective analysis of all data collected during the experiment. The cumulative interaction between the instructional activity design and the empirical data collected, the relationship between the two must be unraveled to produce an optimal learning sequence. Therefore, adaptation must be done when activities that are lacking or not functioning, are abandoned. The activities in the first stage that were carried out and analyzed then provided recommendations for improving learning tools and improving activities in the second stage. The findings from the results of data analysis in the second stage provide the final recommendation of this study.

This research specifically aims to develop local instructional theory on function materials with the help of ICT to develop algebraic thinking skills, which goes through stages (1) development of instructional design or learning trajectory hypothesis – hypothetical learning trajectory (HLT); (2) implementation of instructional design in the classroom; and (3) analysis of student learning process development when involved in design implementation, as well as improvement of instructional design based on implementation results. The results of this study, apart from being an instructional design product, also contain a review of how this design can be used in the classroom using a realistic mathematical approach, as well as students' thinking processes in developing their algebraic thinking skills on function material.

RESULT AND DISCUSSION

As stated in Table 1, the preliminary study's findings were used to create student learning activities that included a hypothetical learning trajectory (HLT) for learning function. The HLT is made up of three parts: student learning objectives, planned instructional activities, and learning media; and the supposed learning process, which predicts how students' thinking and understanding will grow when instructional activities are implemented in the classroom. This HLT is flexible and can be changed during the teaching process.

TABLE 1. HLT for learning function

Learning objectives:

- Students can define relation
- Students can express relation various representations

Learning activities:

- Meetings are held online via the Zoom application.
- Documentation of teaching materials and worksheets can be accessed by students through Google Classroom.
- The teacher reminds again about the concept of the set.
- Students are asked to make a family tree chart.
- Students are asked to find information about the number of wheels from several land transportation means.
- Students are asked to explain the relationship between the two sets formed.
- Students are asked to pair the members of the set based on the given relation.
- The teacher guides students to state the relationships contained in these relationships in a set of consecutive pairs, arrow diagrams, and Cartesian diagrams.
- The teacher guides the students to conclude the definition of the relation.

Allegations of students' way of thinking:

- Recall the concept of the set.
- Create a family tree chart.
- Obtaining information that the relationship that occurs from family relationships may vary.
- Obtaining information that the relation from the set of means of transportation to the number of wheels will be different from the relation from the set of the number of wheels to the set of means of transportation.
- Presenting data obtained from observations in a familiar representation (eg in the form of tables, lists with arrows or colons) in representing the relationships that occur.
- Observing the similarities and differences of several relationships that are formed.

E-ISSN: 2621-4296

Learning objectives:
- Students can define function
- Students can express function various representations
- Students can distinguish between functions and non-functions.
- Students can identify the elements of a function.

Learning activities:
- Meetings are held online via the Zoom application.
- Documentation of teaching materials and worksheets can be accessed by students through Google Classroom.
- Students in groups carry out activities to measure height and weight.
- Students are asked to present the data in various forms of representation.
- Students are asked to explain the properties of the relationships formed.
- The teacher guides students to conclude the concept of the function contained in the relationship

Allegations of students' way of thinking:
- Record the height and weight of each group member.
- Obtain information that each member has a height and weight.
- Obtaining information that it is impossible for a person to have more than one height/weight.
- Presenting data obtained from observations in the form of a table set of ordered pairs, Cartesian diagrams, or arrow diagrams.
- Observe the properties of the relations formed and relate them to the concept of function.

Learning objectives:
- Students can analyze the pattern of relationships of problems related to functions.
- Students can formulate function

Learning Activities:
- Meetings are held online via the Zoom application.
- Documentation of teaching materials and worksheets can be accessed by students through Google Classroom.
- Students are asked to prepare a container to collect water that drips from the water faucet.
- Students record the water level in the container for a certain time interval.
- Students are asked to pay attention to the pattern formed, then formulate a function that maps time (in seconds) to the water level (in mm or cm).
- The teacher demonstrates the use of Geogebra to represent the data in graphical form.

Allegations of students' way of thinking:
- Presenting water level data in the form of graphs, histograms, or bar charts.
- Recheck the measurement scale, both on the horizontal (X-axis) and vertical (Y-axis) axis.
- Recheck the starting point (point of origin) on the graph/diagram created.
- Explore the use of Geogebra in representing the given data.
- Noting that from time to time the water level continues to increase.
- Predicting the next time the water level will increase in height following a certain pattern.

Learning objectives:
- Student can graph a function.
- Students can determine the value of a function

Learning Activities:
- Meetings are held online via the Zoom application.
- Documentation of teaching materials and worksheets can be accessed by students through Google Classroom.
- The teacher gives problems in the form of phenomena in everyday life to obtain data that will form functions. For example parking rates, mileage, prices of various commodities, etc.
- Students are asked to formulate a function from the data obtained.
- The teacher asks students to explore the function (manually as well as with the GeoGebra application).
- Students draw a graph of a function.

Allegations of students' way of thinking:
- Analyzing the pattern formed to formulate the intended function.
- Create tables or ordered pairs based on the function.

E-ISSN: 2621-4296

- Move the obtained points into Cartesian coordinates.
- Explore functions with GeoGebra software.
- Create a graph of a given function.

The learning design that has been developed is implemented in class VIII of one junior high school in Jakarta which consists of 29 students. When the research was carried out, Jakarta was still in the period of Enforcement of Community Activity Restrictions (PPKM) so that learning was carried out online. Online learning is carried out synchronously through the Zoom application and asynchronously through Google Classroom. Synchronous online learning through the Zoom application is carried out in 6 meetings with a time allocation of 2 x 30 minutes in each meeting. While Google Classroom is used by students to download assignments and worksheets and upload the results of working on assignments given by the teacher.

Learning at the first meeting aims to make students able to understand the meaning of relationships, state relationships in various ways, and mention examples of relationships in everyday life. At the beginning of the lesson, the teacher presents a context that is close to the daily lives of students, namely about favorite snacks. Through this context, students are guided to be able to build formal knowledge about the concept of relations. Students are also introduced to the Geogebra application to explore the concept of relations.

Learning at the second meeting aims to make students able to understand functions, state functions in various ways, and distinguish between functions and non-functions. At the beginning of the lesson, the teacher presents a context that is close to the daily lives of students. Through this context, students are guided to be able to build formal knowledge about the concept of function. Students are also introduced to the Geogebra application to explore the concept of function.

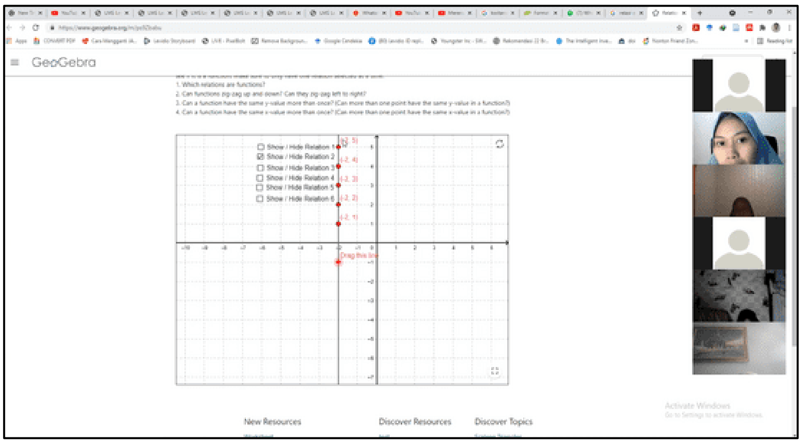

FIGURE 1. Using the Geogebra to explore the concept of function

In addition, students are also asked to work on worksheets that can help students achieve formal knowledge about the concept of function.

FIGURE 2. Students are asked to do activities on the worksheet

E-ISSN: 2621-4296

Learning at the third meeting aims to make students able to formulate functions. At the beginning of the lesson, the teacher presents a context that is close to the daily lives of students, namely about public transportation fares. Through this context, students are guided to be able to formulate functions.

FIGURE 3. Teacher present a context at the beginning of lesson

The teacher guides students to analyze the pattern of relationships that occur in the given context as shown at Figure 4.

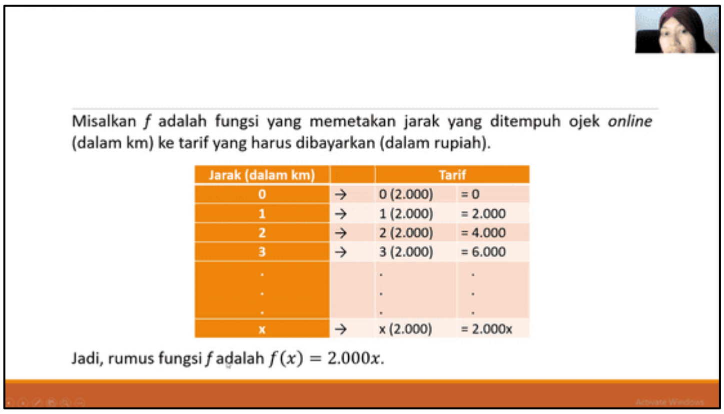

FIGURE 4. Analyzing relationship patterns to formulate functions

Learning at the fourth meeting aims to make students able to draw graphs of functions. The teacher demonstrates how to draw graphs manually and with the help of Geogebra. Students explore problems and draw the required graphs, either manually or with the help of Geogebra as shown at Figure 5. In addition, students are also asked to work on worksheets about graphs of functions.

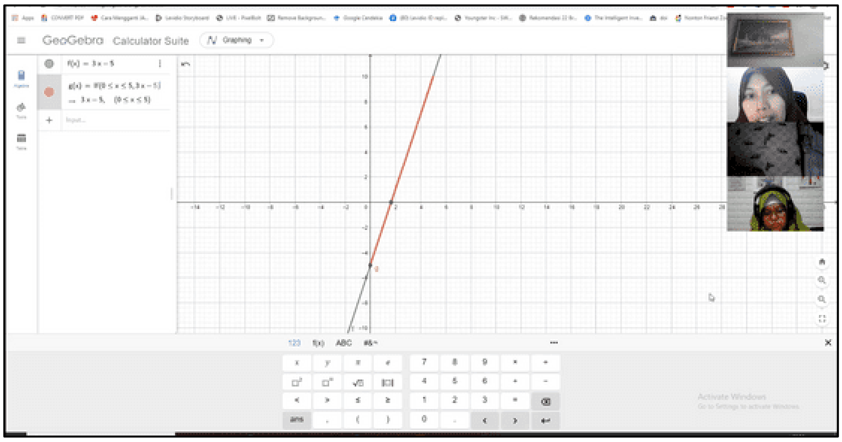

FIGURE 5. The teacher demonstrates how to draw graphs using Geogebra

Learning at the fifth meeting aims to make students able to determine the value of the function. The teacher demonstrates how to determine the value of a function manually and with the help of Geogebra. Students explore the problem and determine the value of the requested function, either manually or with the help of Geogebra. At the sixth meeting, the teacher gave problems about algebraic thinking skills to students related to function material.

This study uses Kieran's (1989) algebraic thinking activities as the foundation for describing students' algebraic thinking skills. This is due to the fact that the viewpoint already incorporates the views of other experts on algebraic thinking skills. Table 2 shows the indicators for each action in algebraic thinking skills.

TABLE 2. Indicators of algebraic thinking skills

Activity	Indicators
Generational	• Students can assess situations using relationship patterns.
Transformational	• Students can switch between various representations. • Students are able to solve mathematical problems using symbolic, visual, or spatial notations, as well as words or sentences.
Global Meta-Level	• Students can investigate issue solving in relation to different topics or branches of science, as well as model and solve the problem.

Students' answers to the problem of algebraic thinking skills about function material have been analyzed. The indicators of each algebraic thinking activity were achieved well by the students.

CONCLUSION

The Realistic Mathematics Education method with the use of ICT is one of the learning ways that may increase students' algebraic thinking skills in the learning function. Understanding the notion of relations and functions, representing functions, creating functions, drawing function graphs, and finding the value of a function are among the learning objectives in hypothetical learning trajectory (HLT) have been developed in this study. The use of HLT has been shown to help students improve their algebraic thinking skills.

REFERENCES

Bora A and Ahmed S 2019 School students’ algebra learning: approaches and barriers Int. Research J. of Humanities, Engineering, & Pharmaceutical Sciences **2 (18)** 74–80

Eisenberg T 2002 Function and associated learning difficulties *Mathematical Thinking* ed D Tall (New York, Boston, Dordrecht, London, Moscow: Kluwer Academic Publisher), pp 140-152.

Feretti F 2019 The manipulation of algebraic expressions: deepening of a widespread difficulties and new characterization *Int. Electronic J. of Mathematics Education* **15 (1)**

Gravemeijer K 1999 How emergent models may foster the constitution of formal mathematics *Mathematical Thinking and Learning* **1 (2)** 155-177

Jupri A and Drijvers P 2016 Student difficulties in matematizing word problems in algebra *Eurasia J. of Mathematics, Science & Technology Education* **12 (9)**

Kalchman M and Koedinger K R 2005 Teaching and learning functions *How Students Learn*

Mathematics in the Classroom ed M Donovan, Suzanne and J D Bransford (Washington DC: The National Academy Press) *https://www.nap.edu/catalog/11101/how-students-learn-mathematics-in-the-classroom2005*.

Kieran C 1989 The early learning o algbra: a structureal perspective *Research Issues in the Learning and Teaching of Algebra* ed S Wagner and C. Kieran pp 35–56

Lew H C 2004 Developing algebraic thinking in early grades: case study of korean elementary school mathematics *The Mathematics Educators* **8**

Michelsen C 2006 Function: a modelling tool in mathematics and science *ZDM* **38 (3)**

Radford L 2006 Algebraic thinking and the generalization of patterns: a semiotic perspective, ed S Alatorre, J L Cortina, M Sáiz and A Méndez *Proc. of The 28th annual meeting of the North American Chapter of The Int. Group for The Psychology of Mathematics Education* (México: Universidad Pedagógica Nacional)