



Exploring the Characteristics of Student Transformational Activities in Indonesia in Solving Cognitive Conflict-Based Problems

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Abstract

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This research aimed to describe the characteristics of the transformational activities of Indonesian students in resolving problems based on cognitive conflict. The research employed a phenomenological approach with five students as subjects. The research instruments included tests and interview guidelines, and the analysis was conducted using the constant comparative method. Change-based activities are a good way to solve cognitive conflict issues, including full rule-based unexpected results, full rule-based matching problem parts, incomplete rule-based unexpected results, and incomplete rule-based matching problem parts. Exploring these characteristics serves as a foundation for instructors to enhance the quality of instructional design. Practically, a profound understanding of these characteristics can provide a solid foundation for educators to enhance their instructional design quality. By exploring and comprehending how students address cognitive conflict, instructors can design more effective learning experiences that align with the student's needs.

Keywords:

transformational activities, cognitive conflict-based problems, missing a fragment

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INTRODUCTION

In the last five years, research has been conducted on solving algebraic problems based on cognitive conflict. According to Setiawan et al. (2023), cognitive conflict related to errors in thinking was found in the construction of mathematical concepts. Another study focused on solving controversial mathematical problems resulting in cognitive conflict (Walida et al., 2022). Cognitive conflicts resolved by students could transform construction errors into accurate constructions (Wyrasti et al., 2018). Cognitive conflict effectively overcame thinking difficulties, achieving an 88.6% success rate (Sumadji & Yuwono, 2020). Research analyzed the implementation of scaffolding based on the cognitive conflict in correcting students' errors in algebraic materials was conducted by Maharani & Subanji (2018). Additionally, a study found that cognitive conflict could minimize students' misconceptions about algebra (Irawati et al., 2018). Based on these several studies, there is a gap in research regarding transformational activities in solving problems based on cognitive conflict that has yet to be explored.



Solving problems based on cognitive conflict is a process of finding problem solutions that generate cognitive conflict (Fraser, 2007; Fujii, 1987; Yang, 2010; Halimah et al., 2019; Pratiwi et al., 2019, 2022; Sela & Zaslavsky, 2007; Sutopo et al., 2019; Walida et al., 2022; Wyrasti et al., 2017, 2018, 2019). Types of algebraic problems that trigger cognitive conflict include problems with equations with no solution or infinite solutions (Fraser, 2007). The potential for cognitive conflict is stimulated by questions and new information containing 'procedural traps.' According to Sela (2008) and Sela & Zaslavsky (2007), when providing challenging procedural knowledge questions, students are confronted with a contradiction due to differences in results obtained with their peers. Similarly, according to Limón (2001), using analogies and metaphors discussed with friends or groups. Each of these problems needs a solution. Students need to learn to solve problems based on cognitive conflict because such solutions are characteristics of the development of cognitive conflict management methods that promote cognitive development (Adey & Shayer, 2002; Fraser, 2007; Toshiakira Fujii, 1987; Niaz, 1995; Watson, 2002).

Unfortunately, Indonesian students still experience situations that lead to cognitive conflict in transformational activities. Cognitive conflict situations arise when students make several errors in solving problems. For instance, errors in thinking while constructing the concept of algebraic square roots, misplacing concepts (misplacement), engaging in pseudo-thinking, and misanalogy (Setiawan et al., 2023). According to Wyrasti et al. (2018), completing assignments involves misanalogy constructions in set theory, fractions, and sequences. Students make mistakes in solving problem-solving questions (Sutopo, 2014). According to Pratiwi et al. (2022), routine errors occur in elementary school teacher education students when solving fraction problems, specifically errors in problem-solving procedures with the same denominator. Another mistake is understanding unresolved problems (Walida et al., 2022). Errors in determining the solution to linear equations with one variable and linear inequalities with one variable, designed with the disappearance of x , occur because students do not understand the meaning of the solution (Fraser, 2007; Fujii, 2003; Fujii, 1987).

The researcher's findings in the preliminary study for the odd semester of 2022-2023 in the Mathematics Education program at UIN Maulana Malik Ibrahim Malang, East Java, further support the phenomenon. A total of 15 students experienced cognitive conflict, three students did not experience cognitive conflict, and one student avoided cognitive conflict. Among the students experiencing cognitive conflict, 40% exhibited signs of four psychological constructions with the potential for destructive cognitive conflict. The remaining 60% of students displayed three signs of cognitive conflict, with the potential for meaningless cognitive conflict. Finding contradictions, being interested, feeling anxious, and reassessing were all signs of cognitive conflict that were looked at (Lee & Byun, 2012). The three types of cognitive conflict that could happen are constructive, destructive, and meaningless (Fraser, 2007; Lee & Kwon, 2001; Sutopo et al., 2019).

The constructive cognitive conflict category indicates that students can describe and overcome conflict, resulting in meaningful understanding. Destructive cognitive conflict suggests that students attempt to address the conflict but have not

yet found a solution to the problem. Meaningless cognitive conflict involves students experiencing cognitive conflict without realizing it and not finding a solution. Here is an example of a response from a student who experienced a cognitive conflict situation with the potential for destructiveness during the final exam, as seen in Figure 1.

The figure shows a student's handwritten work for solving the equation $\frac{1}{3}(12 - 6x) = 4 - 2x$. The work is annotated with three boxes:

- missing a fragment**: A bracket on the right side groups the first three lines of work.
- an unexpected result**: A box on the left side points to the line $4 - 4 = -2x + 2x$.
- matching problem components**: A box at the bottom points to the line $0 = 0$.

The handwritten work consists of the following lines:

$$\frac{1}{3}(12 - 6x) = 4 - 2x$$

$$4 - 2x = 4 - 2x$$

$$4 - 4 = -2x + 2x$$

$$0 = 0$$

Figure 1. EAS answers

Based on the results of the EAS work, the situation led to cognitive conflict, manifesting as missing a fragment, an unexpected result, and matching problem components. In this situation, several students were observed solving problems related to linear equations with one variable based on rules (rule-based). Such problem-solving activities are called transformational activities (Kieran, 2004, 2006; Kilpatrick et al., 2001; Palatnik & Koichu, 2017). The transformational activity in this research involves solving equations to maintain equivalent equations (Kieran, 2004). According to Malloy et al. (2002), solving equations to maintain equivalent equations involves procedural steps such as writing equations, using the distributive property, applying the equality property, simplifying, and determining the solution. Transformational activity is an integral part of algebraic thinking. Algebraic thinking is a mental process involving generational, transformational, and meta-global-level activities related to algebraic and non-algebraic subjects (Apsari et al., 2019; Carraher et al., 2008; Ellis, 2011; Kieran, 2004, 2018; Lannin, 2005; Levin & Walkoe, 2022; Strachota et al., 2018; Verschaffel et al., 2018).

Although some students' answers have involved algebraic thinking through transformational activities, they still need to be completed. For instance, a student may use an equivalent equation to the correct answer but fail to determine the solution to the given problem or use an equivalent equation to the wrong answer. Each student's response possesses specific characteristics of their transformational activity. According to the Cambridge Dictionary, the specific qualities of an individual are referred to as characteristics. In this context, characteristics are the distinctive features of transformational activities in solving problems based on cognitive conflict that are described in detail (Barile, 2023). These characteristics help individuals recognize transformational activities, making it easier to describe the details of these activities clearly. The characteristics are built by examining transformational activities in solving problems based on cognitive conflict. This

examination involves observing the indicators of each transformational activity and the situations in solving algebraic problems that lead to cognitive conflict.

The suspected characteristics that will emerge from transformational activities in solving single-variable linear equation problems based on cognitive conflict are based on the phenomena observed in the preliminary study. In the first group, students engaged in incomplete rule-based activities with correct answers exhibiting various situations (missing a fragment, violation of a rule, an unexpected result, matching problem components) without realizing that one procedure only sometimes applies to both cases. The situations also included instances where students were unaware that one procedure only sometimes applies to both cases (missing a fragment, an unexpected result, matching problem components). In the second group, students performed incomplete rule-based activities with incorrect answers, featuring various situations (missing a fragment, a rule violation, an unexpected result, matching problem components) without realizing that one procedure only sometimes applies to both cases. Students encountered these variations of situations that had the potential for destructive and pointless cognitive conflict.

In these characteristics, the emergence of characteristics where students work based on complete rules with correct answers has yet to be apparent, and situations involving matching problem components, where students review the problem and apply suitable rules, have yet to emerge. The absence of these characteristics can be interpreted as a need to achieve the main focus of learning. This is because solving problems is the primary focus of learning (Jonassen, 2010; Susiana, 2018). The solution obtained in solving a problem, incorporating existing knowledge, indicates engaging in thinking activities (Kieran, 2004). Therefore, solutions to algebraic problems need to be determined. The solution to an equation is any substitution of variables that makes the equation true, and solving an equation means finding all its solutions (Bittinger & Beecher, 2012). The absence of these characteristics has prompted researchers to explore the characteristics of transformational activities in solving conflict-based cognitive problems. This study aims to describe the characteristics of the transformational activities of Indonesian students in solving problems based on cognitive conflict.

METHODS

Participants

This research applied a qualitative approach with a phenomenological design to describe the characteristics of transformational activities in solving problems based on cognitive conflict. The study was conducted in the odd semester of 2023-2024 and involved 143 students from five universities in Indonesia. These universities include the Mathematics Education Study Programmes at IAIN Lhokseumawe and IAIN Takengon in Aceh, UIN SATU Tulungagung in East Java, the University of Lambung Mangkurat in South Kalimantan, and STKIP YDB Lubuk Alung in West Sumatra. Out of 143 respondents, 10 data points were excluded due to errors, as the questions provided were ambiguous and "the exclamation mark (!) at the end of the question was assumed to mean factorial," and

there was a mistake in writing a number in the question; it should have been 15 but was written as 5. Thus, the data were reduced to 133 respondents. These students had completed courses in calculus or elementary algebra.

The research instruments included a test on algebra problems based on cognitive conflict and an interview guide. The test on algebra problems based on cognitive conflict was used to describe situations that raise individuals' awareness of the mismatch between their concepts and their environment. Meanwhile, the interview guide was used to clarify the obtained data and identify the correspondence between students' written answers and oral explanations in solving problems based on cognitive conflict. In this study, to clarify situations that cause cognitive conflict in the categories of missing a fragment, violation of a rule, an unexpected result, and matching problem components (Gal, 2019b), potential subjects were chosen from respondents showing signs of cognitive conflict in four psychological components: (a) recognition of contradiction; (b) interest; (c) anxiety; and (d) reassessment (Lee & Byun, 2012), with the potential for cognitive conflict falling into the categories of constructive, destructive, and meaningless. Of the 133 respondents, 106 experienced cognitive conflicts, while 27 did not. The 106 respondents selected research subjects based on cognitive conflict categories (a, b, c, and d) and (b, c, and d), assuming both have the same psychological construction, resulting in 61 potential research subjects.

The study focused on potential subjects who solved problems based on cognitive conflict with complete and incomplete transformational activity results. This means potential subjects determined the solution to the problem using complete and incomplete equivalent equations, totaling 19 potential subjects. These potential subjects were obtained from the potential cognitive conflict group of 17 constructive and two destructive potential subjects. Therefore, there were 19 potential subjects to explore the characteristics of their transformational activities. \

Data collection

Interviews and recordings of students' activities in resolving cognitive conflict-based problems served as the basis for data collection. The data identified various student responses to solving problems based on cognitive conflict based on indicators of transformational activities, specifically solving single-variable linear equations based on rules (rule-based). This means that students used equivalent equations and determined the solutions to algebraic equations. These activities were combined with indicators of situations in mathematics that led to cognitive conflict, namely: (1) Missing a fragment, where a missing procedure was identified; (2) Violation of a rule, where a procedure that did not comply with the rules was found; (3) An unexpected result, where an unexpected outcome was found; (4) Matching problem components, which include (a) not realizing that a known procedure cannot be used and (b) students reviewing the problem and using suitable rules. The results of the combination of these two indicators were: (1) solving single-variable linear equations based on rules with the discovery of a missing procedure; (2) solving single-variable linear equations based on rules with the discovery of a procedure that violates the rules; (3) solving single-variable linear equations based on rules with the discovery of an unexpected result; and (4) solving single-variable

linear equations based on rules with (a) not realizing that a known procedure cannot be used and (b) students reviewing the problem and using suitable rules.

The combination of these indicators serves as the basis for several potential characteristics of transformational activities among Indonesian students in solving problems based on cognitive conflict. However, these assumptions may change based on the results of the research data analysis. Research subjects were selected using purposive sampling, considering potential subjects could communicate effectively when asked for further information regarding the completed work process. Five of the 19 selected potential subjects were chosen: DW, SNR, EY, DR, and NP.

Subjects DW, SNR, and EY each represent the group that worked based on complete rules, with situations involving an unexpected result and matching problem components. Subject DR worked based on incomplete rules, with situations involving a missing fragment, an unexpected result, and matching problem components. Subject NP worked based on incomplete rules, with situations involving a violation of a rule, an unexpected result, and matching problem components.

The collected data, in the form of answer sheets and interview transcripts, has been validated using triangulation methods. Subsequently, the data was analyzed using a qualitative analysis technique, namely the constant comparative method. According to Glaser and Strauss (1967) (Kolb, 2012), the technical analysis of the constant comparative method involves four stages: (i) Comparing incidents applicable to each category; (ii) Integrating categories and their properties, which involves coding transformational activity and situations leading to cognitive conflict; (iii) Delimiting the theory; and (iv) Writing the theory. This is done by consistently comparing one data set with another and then systematically comparing categories. The results of this analysis are used to describe algebraic thinking in transformational activities when solving problems based on cognitive conflict, taking into account situations in mathematics that lead to cognitive conflict, namely: (1) Missing a fragment; (2) Violation of a rule; (3) An unexpected result; and (4) Matching problem components.

RESULTS & DISCUSSION

This study reveals the characteristics of transformational activities among Indonesian students in solving problems based on cognitive conflict. These characteristics were derived from analyzed answer sheets and interview data through the constant comparative method. The integration of codes from transformational activities and situations leading to cognitive conflict yielded transformational characteristics for each subject, namely DW, SNR, EY, DR, and NP, as depicted in the following figures:

Figure 3 describes the answer sheet and interview results of subject DW in solving problems with transformational activities. Although the subject did not write the concept's name on the answer sheet, such as the distributive property, when confirmed through the interview, subject DW mentioned and used the concept of the distributive property in their work. The subject also experienced a cognitive conflict.

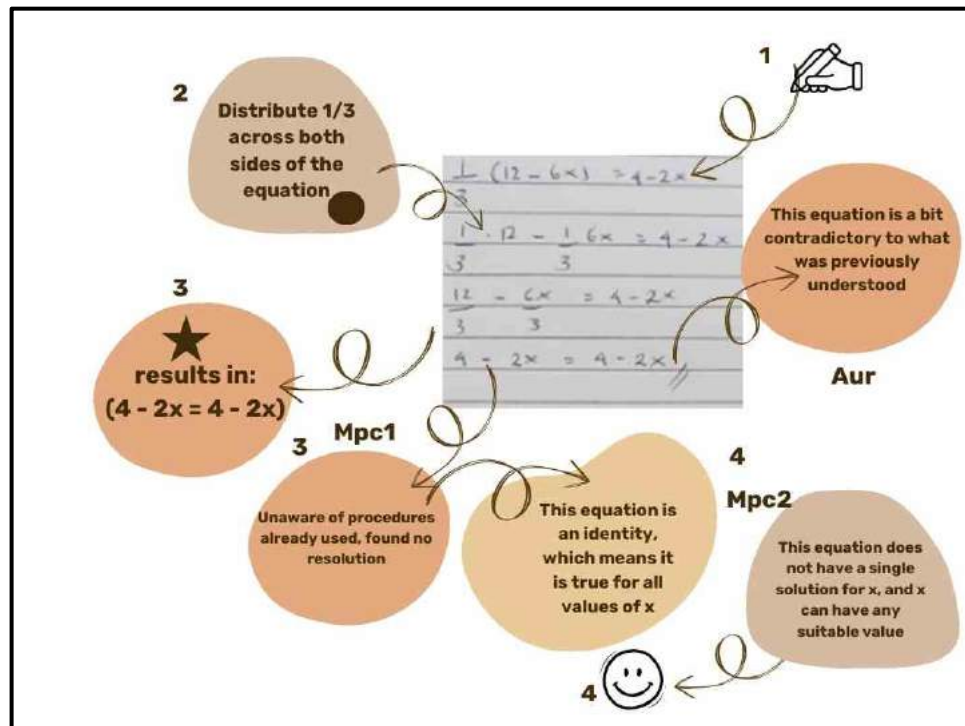


Figure 2. DW Transformational Activities In Completing Cognitive Conflict-Based Problems

The results from Figure 2 show that the characteristics of subject DW involve transformational activities with stages (writing the equation, using the distributive property, simplifying, and determining the solution). The situations leading to cognitive conflict include finding unexpected results (an unexpected result) and not realizing that a known procedure cannot be used, as well as applying a known procedure using matching problem components. When subject DW encountered an unexpected result, they reconsidered a suitable solution for the given problem. The interview revealed that subject DW's thought process was as follows: *"I noticed that both sides of the equation have identical forms, which means there is no x on one side of the equation and none on the other side. This indicates that this equation is an identity, which means it is true for all x values. Therefore, I concluded that the solution to this equation is x can have any value or can be expressed as 'unlimited solutions' or ' x is a free variable.' So, this equation does not have a single solution for x , and x can take any value that fits."*

This thought process indicates that the equation's solution is all real numbers. Similarly, subject SNR also engaged in transformational activities. The steps used to maintain equivalent equations are clarified in their answer sheet. Cognitive conflict situations also occurred for subject SNR. Transformational activities in solving problems based on cognitive conflict can be seen in the following Figure 3.

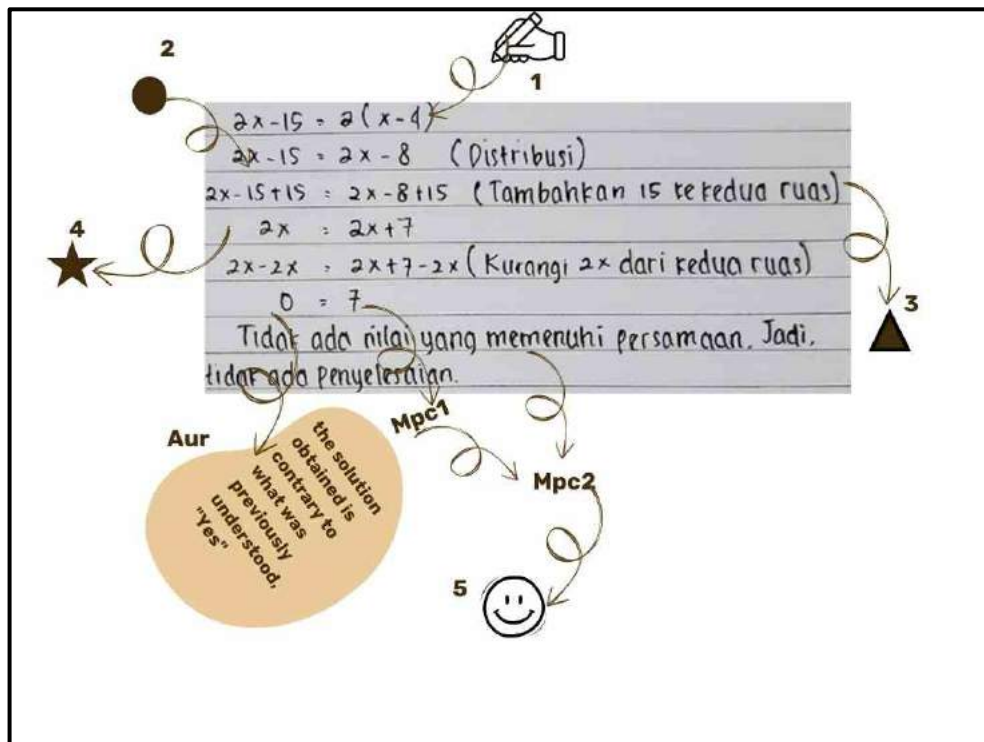


Figure 3. SNR Transformational Activities In Solving Cognitive Conflict-Based Problems

The results in Figure 3 show the features of the subject SNR, which led to transformational activities that happened in stages (writing the equation, using the distributive property, using the equality property, making things easier, and finding the answer). The cognitive conflict encountered includes an unexpected result and a lack of awareness that known procedures cannot be used, as well as applying known procedures using suitable rules (matching problem components). When subject SNR encountered an unexpected result, they rethought the problem. In the interview, subject SNR stated, "I recalculated repeatedly to make sure there was no mistake because the values of x were exhausted, and there was no x value that satisfied it, so there was no solution." Subject SNR's response indicates an inconsistent equation; its solution set is the empty set (\emptyset). Next, subject EY engaged in transformational activities without using the distributive property in the answer sheet and interview but instead utilized the equality property. The subject stated, "Multiply both sides by the number 3." Subsequently, a cognitive conflict occurred for subject EY. Figure 4 illustrates the transformational activities in solving conflict cognitive-based problems for subject EY.

The indicates that subject EY's characteristics involve engaging in transformational activities with the following steps: writing the equation, using the equality property, simplifying, and determining the solution. An unexpected result was encountered in the given situation, and the subject was unaware that a known procedure could not be used. However, the subject applied a known procedure using suitable rules (matching problem components). When subject EY encountered an unexpected result, they reconsidered the problem. According to the interview results, subject EY stated, "It turns out that the problem has many solutions, so the lines formed are overlapping."

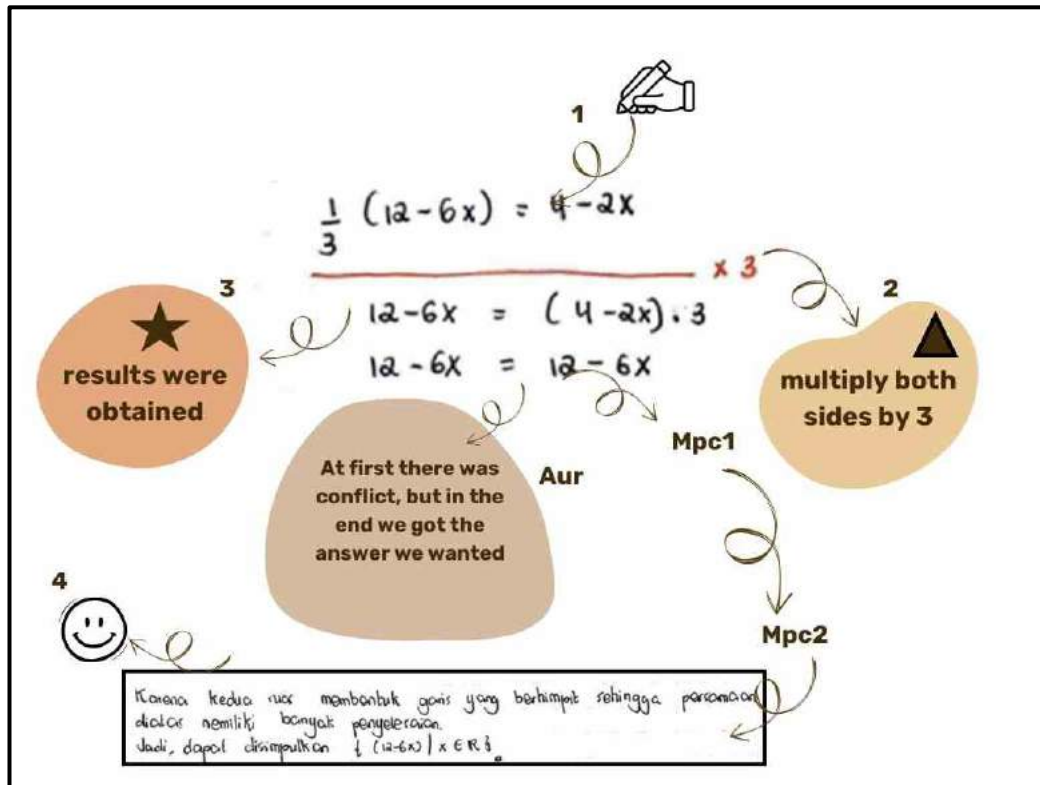


Figure 4. EY Transformational Activities In Completing Cognitive Conflict-Based Problems

The subject's thought process indicates that the solution set for the given equation includes all real numbers. Subject DR also engaged in transformational activities, experiencing situations in solving problems based on cognitive conflict, as presented in Figure 6 below. The answer sheet and interview needed more information about the procedural steps.

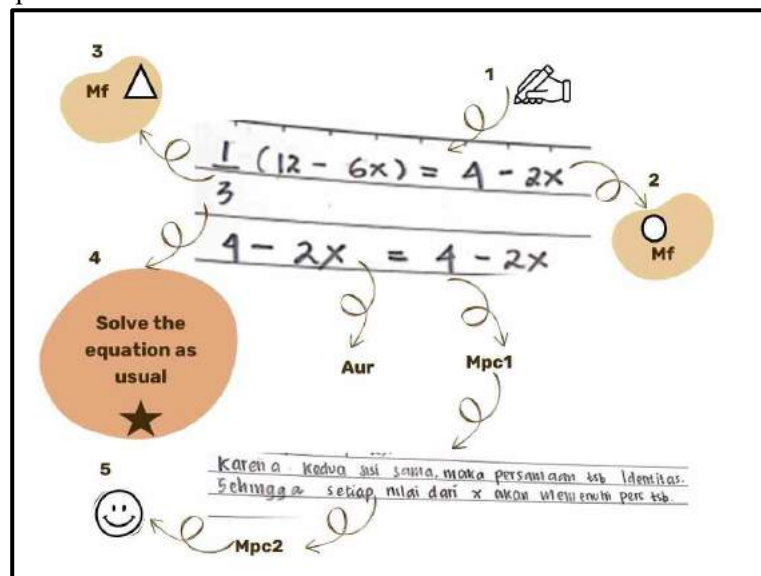


Figure 5. DR Transformational Activities In Completing Cognitive Conflict-Based Problems

Figure 5 shows that the characteristics of Subject DR involve transformational activities with the following steps: writing the equation, not using the distributive property, not using the equality property, simplifying, and determining the solution. The situation encountered includes finding a missing procedure (missing a fragment), encountering an unexpected result (an unexpected result), and not realizing that a known procedure cannot be used, along with applying a known procedure by using suitable rules (matching problem components). When Subject DR encountered an unexpected result, they reconsidered the solution. The interviewee mentioned, "The equation is solved by multiplying 3 on both sides. Because both sides are the same, the equation is an identity, so the value of x will satisfy the equation." According to the result that Subject DR expressed, the given equation's solution set contains only real numbers.

The transformational activities and situations encountered in solving problems based on cognitive conflict performed by Subject NP are presented in Figure 6 below. Subject NP did not write the distributive property in the transformational activity stage. Still, when confirmed through an interview, it was revealed that they used the distributive property.

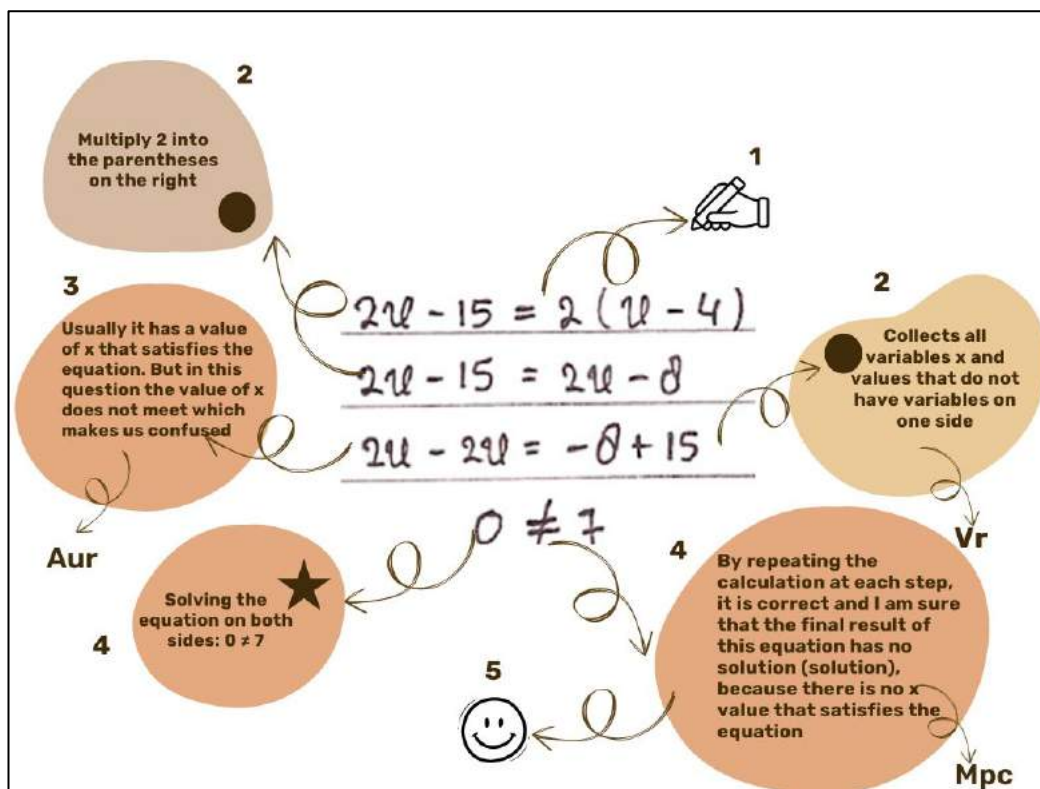


Figure 6. NP Transformational Activities In Solving Cognitive Conflict-Based Problems

Figure 6 shows that Subject NP's characteristics involve transformational activities with the following steps: writing the equation, using the distributive property, not using the equality property, simplifying, and determining the solution. The situations encountered include finding a procedure that violates the rule (violation of a rule), encountering an unexpected result (an unexpected result), and not realizing that a known procedure cannot be used, along with applying a known procedure by using suitable rules (matching problem components). When Subject NP encountered an unexpected result, they reconsidered the solution. In the interview, it was stated, "By repeating the calculation to check for errors at each step, if it is correct, I am confident that the final result of this equation has no

solution or can be said to be a contradictory equation because there is no value of X that satisfies the equation." Subject NP's response indicates an inconsistent equation; the solution set is empty (\emptyset). The explanation of the codes in several figures above can be seen in Figure 7.

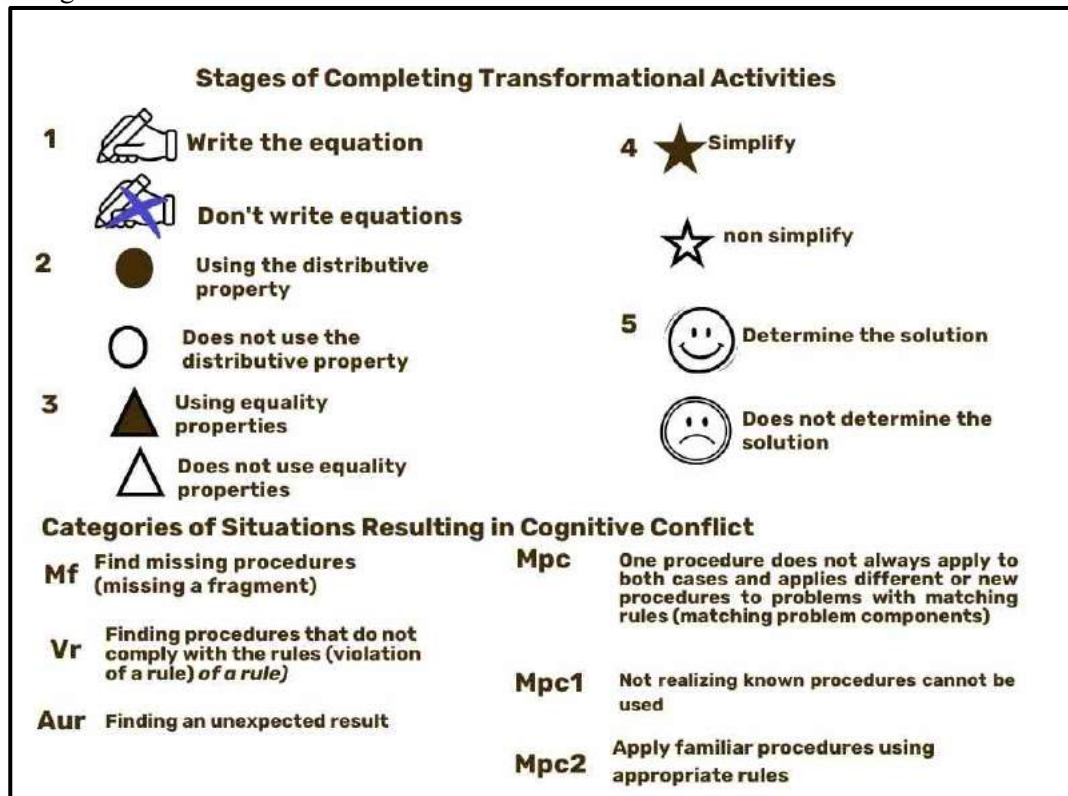


Figure 7. Codes From Transformational Activities And Situations That Result In Cognitive Conflict

After obtaining the characteristics of transformational activities in solving conflict-based cognitive problems carried out by subjects DW, SNR, EY, DR, and NP, they were compared to determine the intersection of these characteristics. The intersection of subjects DW and SNR resulted in the characteristics of transformational activities for writing the equation, using the distributive property, simplifying, and determining the solution, with situations leading to cognitive conflict in the categories of an unexpected result and matching problem components. The intersection of subjects SNR and EY produced the characteristics of transformational activities for writing the equation, using equality properties, simplifying, and determining the solution with situations of an unexpected result and matching problem components. Meanwhile, the intersection of the characteristics of subjects DR and NP yielded the characteristics of transformational activities: writing the equation, simplifying and determining the solution in situations of unexpected results, and matching problem components.

CONCLUSION

Based on the investigation and analysis, it can be concluded that the transformational activities of Indonesian students in solving problems based on cognitive conflict can be characterized into two groups: complete rule-based

transformational activities, with situations of an unexpected result and matching problem components, and incomplete rule-based transformational activities, with situations of an unexpected result and matching problem components. Therefore, the characteristics of transformational activities of Indonesian students in solving problems based on cognitive conflict are complete rule-based unexpected results, complete rule-based matching problem components, incomplete rule-based unexpected results, and incomplete rule-based matching problem components.

This exploration of characteristics serves as a foundation for instructors to enhance the quality of instructional design. The findings of this research provide an opportunity for further investigation into the characteristics of other algebraic thinking activities, such as those at generational and meta-global levels. This is necessary to delve deeper into students' algebraic thinking activities in solving problems based on cognitive conflict.

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