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Exploring High School Students' Mathematical Reasoning in Geometry Using GeoGebra-Assisted Problem-Based Learning

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Abstract
The significance of mathematical reasoning in mathematics instruction and the substantial incidence of inadequate mathematical reasoning among high school students prompted this study. This study intends to explore the mathematical reasoning of high school students about three-dimensional geometry using the GeoGebra-assisted Problem-Based Learning (GPBL) model. This research applied a quasi-experimental design including a non-equivalent pretest and posttest control group. The subjects of this research were specifically 12th-grade students from an upper secondary school in Subang. The research contained two classes, with 27 students as the experimental group and 27 as the control group. Interviews were conducted to identify the factors that explain students' difficulties in solving mathematical reasoning problems concerning three-dimensional geometry. The findings indicated that (1) overall, the enhancement of mathematical reasoning among students utilizing the GPBL model was significantly higher than that of students implementing the conventional learning model; (2) referring to students' prior mathematical ability (PMA), the enhancement in mathematical reasoning for students in high, medium, and low PMA groups utilizing the GPBL model was significantly higher than that of students implementing the conventional learning model; (3) the main obstacle discovered by students to solving mathematical reasoning problems involving three-dimensional geometry is their struggle in visualizing problems in a three-dimensional context. GeoGebra assists students in visually constructing three-dimensional geometry problems and evaluating the outcomes of algebraic algorithms with 3D graphical representations.

Keywords: Mathematical Reasoning; Geometry; Problem-Based Learning; GeoGebra; Prior Mathematical Ability

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INTRODUCTION

Mathematics is an essential life capital, particularly in addressing daily issues. Mathematics demonstrates a systematic logic defined by an ordered and precise sequence. Participation in mathematics can enhance children's higher-order thinking abilities (Laurens et al., 2017). Mathematics is a fundamental discipline with extensive practical applications in everyday life and several domains. Mathematics cultivates critical thinking abilities, allowing individuals to assess occurrences, recognize patterns, and devise solutions for problem-solving and decision-making (Zapata et al., 2024). Mathematics is intrinsically connected to mathematical activities, including mathematization, exploration, reasoning, and communication. These procedures indicate sophisticated cognitive capabilities (Wittmann, 2021). Mathematics is a compulsory discipline pursued from primary education through tertiary education. This issue is crucial for advancing research and technology and directly affects human existence (Retnawati et al., 2020).

Considering the properties mentioned in mathematics, educational institutions should create environments that make mathematics study meaningful for students. Meaningful learning transpires when learners actively integrate new material with prior knowledge, enhancing engagement, retention, and knowledge transfer (Andrews et al., 2023). The efficacy of mathematics learning is affected by classroom management, student assistance, cognitive engagement, subject-specific quality, and pedagogical quality (Schlesinger, 2018). The effectiveness of mathematics learning depends on educators' thorough comprehension of the elements that facilitate successful learning, including effective theories, models, processes, methods, and media. The active learning process is defined by students' physical, mental, and emotional engagement (Indrapangastuti et al., 2021). Effective mathematics instruction necessitates linking classroom teaching to real-world applications, meticulously preparing lessons beforehand, delivering clear explanations of mathematical concepts, utilizing explicit examples, assigning adequate homework and practice, conducting assessments with constructive feedback, promoting collaborative group work, and addressing the diverse needs of students (Ukobizaba et al., 2021).

"Principles and Standards for School Mathematics" is an extensive framework to enhance mathematics instruction and comprehension in educational environments, focussing on equity, curriculum, teaching, learning, assessment, and technology. These concepts can impact curriculum design, individual mathematics sessions, educator responsibilities, professional development opportunities, and other elements. "Principles and Standards for School Mathematics" comprises topic and procedural standards delineating our mathematics learning priorities. The content standards encompass numbers and operations, algebra, geometry, measurement, data analysis, and probability, whereas the process standards include problem-solving, reasoning and proof, communication, connections, and representation (NCTM, 2000). The objective of Mathematics Learning in schools, as stated by the Ministry of National Education (Depdiknas), is for students to (1) comprehend mathematical concepts and elucidate their interrelations; (2) employ reasoning to discern patterns and properties, execute mathematical operations to formulate generalizations, collect evidence, or articulate mathematical concepts and propositions; (3) resolve mathematical problems; (4) convey ideas through symbols, tables, diagrams, or other media to elucidate situations or issues; and (5) cultivate an appreciation for the practical applications of mathematics in everyday life (Depdiknas, 2007). Mathematics, meaningful learning, technology, geometry, and mathematical reasoning are essential for attaining mathematics learning objectives.

Mathematical reasoning is an essential element of mathematics learning, and the development of informed learning abilities and innovations is acknowledged as a means to enhance human quality and potential in the 21st century (Oslington et al., 2020). Mathematical reasoning has become an essential component of the mathematics curriculum in numerous countries, mainly due to the emphasis on mathematical process competencies—such as modeling, problem-solving, and reasoning—by the Programme for International Student Assessment (PISA) (Hjelte et al., 2020). Mathematical reasoning emphasizes a systematic and logical approach to thought and deriving new insights from pertinent data and sources (Walkington et al., 2019). Reasoning is crucial for mathematics users as it entails an active, dynamic, and generating process. Reasoning enables students to systematically organize mathematical concepts and comprehend mathematics (Cabello et al., 2021). Figure 1 illustrates that the mathematical reasoning scores of Indonesian students remain insufficient in PISA 2022 (OECD, 2023).

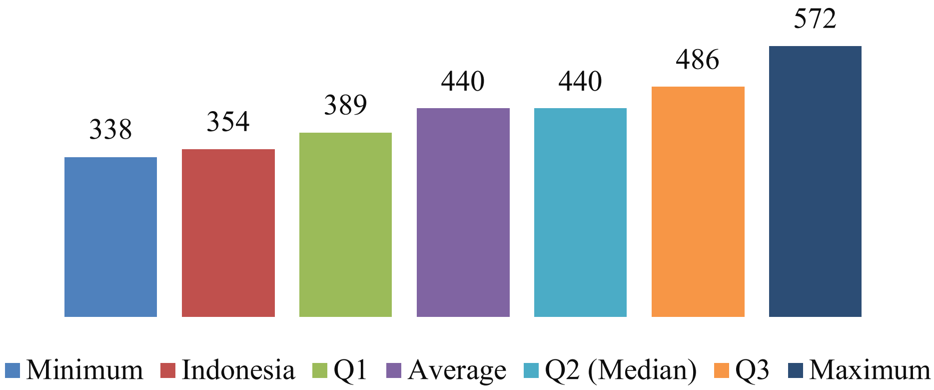

Figure 1. Graph of Indonesian Students' Mathematical Reasoning Score on PISA 2022

The Problem-Based Learning (PBL) model suggests better effectiveness than conventional learning models in enhancing students' mathematical reasoning abilities. The initial phase of problem-based learning (PBL) commences with presenting a problem, prompting students to apply their mathematical reasoning to derive a solution (Sary & Fatimah, 2023). In the educational process utilizing the PBL model, students collect information, assess problems, and formulate solutions. The PBL paradigm is derived from real-world challenges frequently faced by students (Sugiman et al., 2019). Implementing the PBL model in educational contexts significantly impacts students' mathematical reasoning abilities. Mathematical reasoning encompasses the capacity for logical and analytical thought, enabling individuals to conclude deductive and inductive approaches. The PBL model employs a problem-centred approach to education. The identified challenges are associated with students' real-life experiences and can enhance their critical thinking and mathematical problem-solving abilities. The PBL paradigm in educational institutions promotes the enhancement of higher-order thinking abilities in students. Furthermore, it fosters students' active and independent development of problem-solving abilities through engagement in data searches to acquire logical and authentic solutions. Students evaluate that the assigned tasks promote and improve active thinking and learning relative to previous experiences. The PBL paradigm is an alternative educational framework applicable at multiple levels to enhance students' mathematical reasoning (Napitupulu et al., 2016; Sari et al., 2020; Mandasari, 2021; Fitriyah et al., 2022).

Geometry education enhances students' mathematical problem-solving and reasoning abilities (Jupri, 2017). Examining geometry, particularly distance in three dimensions, provides individuals various perspectives and educational experiences. Geometry's visual-spatial orientation directly engages with abstract spatial reasoning techniques, focusing on issue comprehension and information manipulation (Buckley et al., 2019). Figure 2 illustrates that Indonesian students' geometry scores (space and shape) remain insufficient in PISA 2022.

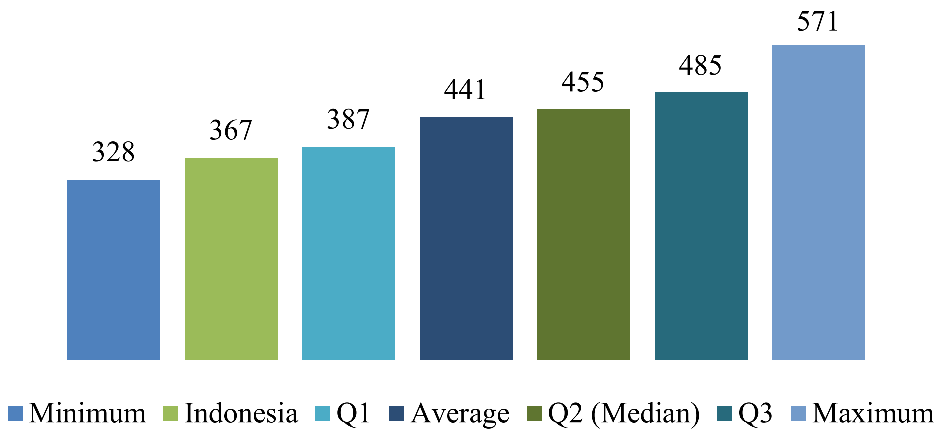

Figure 2. Graph of Indonesian Students' Geometry (Space and Shape) Score on PISA 2022

Spatial ability encompasses a range of functions and capacities that facilitate manipulating and processing spatial information. Spatial aptitude is crucial in STEM fields, as students must visualize objects in various orientations, manipulate three-dimensional models, and mentally convert images from two to three dimensions. Many students face challenges comprehending 3D spatial geometry and shifting from 2D to 3D representations. The capacity to generate precise representations of spatial objects and comprehend three-dimensional visual-spatial relationships significantly impacts an individual's proficiency in learning spatial geometry. Targeted training with suitable instruments can improve spatial abilities (Carbonell-Carrera et al., 2021). Incorporating technology and digital resources in geometry education is advisable, as interactive manipulatives facilitate the transition among concrete, pictorial, and symbolic (abstract) representations. Transitions are crucial for comprehending mathematical concepts, examining relationships, producing precise graphical representations (GR), developing and verifying hypotheses, and employing diverse problem-solving strategies (Žakelj & Klancar, 2022).

Integrating technological advancements into the educational process is crucial for improving student learning outcomes efficiently. Diverse educational models enable educators to leverage technological advancements to enhance students' abilities and competencies (Hermino & Arifin, 2020). The education system must promptly adapt to rapid transformation and technological progress changes, reflecting daily technological advancements (Liburd & Jen, 2021). A correlation exists between computer-based learning mediums and students' spatial abilities in mathematics. Digital learning resources improve students' spatial abilities. Spatial ability is an essential skill that students must cultivate to address challenges in geometry effectively (Kamid et al., 2020). Educators are pivotal in imparting media literacy within the educational framework. Media literacy competency encompasses the information, abilities, and attitudes required to effectively access, analyze, evaluate, produce, and engage with many kinds of communication across different digital media (Fadlillah et al., 2023). GeoGebra serves as a digital educational tool employed in the field of mathematics. GeoGebra is an interactive tool developed for geometry, algebra, statistics, and calculus, intended to improve the teaching and comprehension of mathematics, science, and engineering. The dynamic interface enables precise and interactive visualization of tasks, models, and outcomes for users. GeoGebra encompasses three essential components: modeling, visualization, and programming (MVP) (Ziatdinov & Valles, 2022). The software application aids students in visualizing their thoughts via defined concepts and graphical representations. This software enables students to verify the accuracy of each stage in addressing a complex problem (Zulnaidi et al., 2020). GeoGebra demonstrates the significance of construct manipulation, facilitating experimentation and exploration of geometric concepts, both planar and spatial, and highlights its efficacy as a dynamic and interactive tool (de Sousa et al., 2021).

Several factors affect students' success in mathematics, such as foundational mathematical aptitude, prior mathematical ability, motivation, and problem structure (Siswono, 2018). Mathematical reasoning is crucial for drawing essential conclusions and establishing causal relationships based on individuals' prior knowledge (Dong et al., 2020). Early mathematical abilities significantly predict success in mathematics learning (Braak et al., 2022). Prior mathematical knowledge predicts examination performance, academic achievement, and the ability to navigate challenges in the mathematics learning process. A comprehensive understanding of mathematical concepts in education is a fundamental requirement for learning (Rach & Ufer, 2020). This research explores high school students' mathematical reasoning in three-dimensional geometry through the GeoGebra-assisted Problem-Based Learning (GPBL) model. This study investigates the correlation between prior mathematical ability and the variables influencing students' challenges in solving geometric problems.

This study focuses on the following research questions: (1) Does the GPBL model enhance students' mathematical reasoning more than the conventional learning model? (2) Is the enhancement in students' mathematical reasoning with the GPBL model higher than that of the conventional model when analyzed by Students' Prior Mathematical Ability (High, Medium, Low)? and (3) What factors contribute to students' difficulties in solving mathematical reasoning problems related to three-dimensional geometry?. This research explores (1) the enhancement of students' mathematical reasoning through the GPBL model compared to the conventional learning model; (2) the differential enhancement of mathematical reasoning among students receiving the GPBL model, categorized by their prior mathematical ability (High, Medium, Low), instead of those in the conventional model; and (3) the factors contributing to students' challenges in solving mathematical reasoning problems related to three-dimensional geometry.

METHODS

This study employs a quantitative approach using a quasi-experimental design with a non-equivalent pretest and posttest control group (Creswell & Creswell, 2018). The study population comprises all 12th-grade students at a high school in Subang. This study's sample consists of two classes, with 27 students in the experimental group and 27 in the control group. Sampling was performed by purposive sampling. The objective of sample selection is to guarantee that the research is executed successfully and efficiently, especially in light of the condition of the research participants, the duration of the study, and the research topic. The focus of the study is mathematical reasoning about three-dimensional geometry. The research instruments used in this study are learning and data-collecting tools. This research uses Lesson Plans and Learning Modules as its educational tools. The data-collecting instruments include tests and non-tests. The data-gathering instrument is a mathematical reasoning exam. Conversely, non-test assessments' data collection devices include observation sheets used during learning activities and structured interview procedures. Interviews are used to ascertain the aspects contributing to students' challenges in resolving mathematical reasoning issues in three-dimensional geometry. The researcher used the GPBL model for the experimental group, while the control group utilized the conventional learning (CL) model. Both groups participated in pretests and posttests to evaluate students' mathematical reasoning before and after the learning process. This study was carried out from July 22 to August 24, 2024. The first meeting included a pretest, followed by implementing the GPBL model for the experimental class from the second to the seventh meeting. The CL model was used for the control group. The ninth meeting included a posttest.

The research stages are as follows: (1) analyzing student characteristics; (2) assessing students' prior mathematical ability (high, medium, low); (3) administering a mathematical reasoning pretest; (4) executing the learning process; (5) administering a mathematical reasoning posttest; and (6) analyzing the research data. Data analysis techniques encompass the computation of the mean and standard deviation for the mathematical reasoning pretest and posttest outcomes in both the experimental and control groups, assessing the enhancement in mathematical reasoning via the normalized-gain (N-gain) formula (Hake, 1998), performing normality and homogeneity tests, and calculating the statistics for the two-sample t-test. Figure 3 illustrates the steps of the flowchart for the statistical testing of the study.

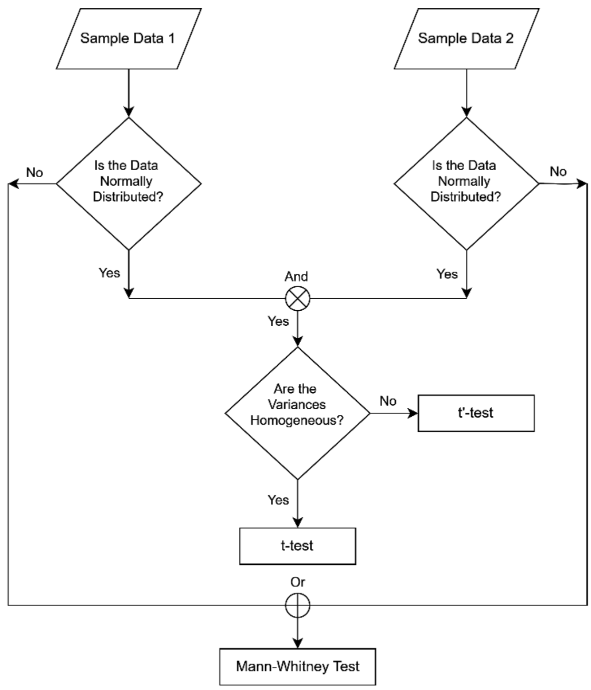

Figure 3. Flowchart of Research Statistical Test (Gunawan, 2017)

It is essential to provide operational definitions to prevent misinterpretations of the terminology used in this work. This study identifies markers of mathematical reasoning as (1) formulating and testing mathematical hypotheses, (2) performing mathematical operations, (3) constructing and assessing mathematical arguments, and (4) logically inferring conclusions or facts (Fisher, 2021). The central strategy in the PBL model is to offer students opportunities to address problems typically encountered within a particular domain. This study outlines the stages of the PBL model: (1) orienting students to the problem; (2) organizing students for study; (3) facilitating independent and group investigations; (4) creating and presenting artifacts and exhibits; and (5) analyzing and evaluating the problem-solving process (Arends, 2012). Consequently, the researcher confines the study topic to three-dimensional geometry related to distance and angles.

RESULTS & DISCUSSION

The enhancement of MR, determined through pretest and posttest outcomes, was assessed using N-gain. Table 1 displays the N-gain data related to students' MR derived from the PMA with GPBL model treatment and the CL model.

Table 1. Descriptive Statistics of Pretest, Posttest, and N-gain Scores

Learning Group	PMA	N	Pretest		Posttest		N-gain	
			Mean	SD	Mean	SD	Mean	SD
Experiment	High	7	63.57	4.76	91.43	6.90	78.32	17.11
	Medium	6	41.15	11.02	70.77	16.81	53.62	21.04
	Low	13	15.71	4.50	35.71	7.87	23.93	5.65
	Overall	27	40.37	19.36	67.04	24.23	52.32	26.06
Control	High	14	63.33	4.08	75.83	5.85	34.82	10.34
	Medium	7	41.43	10.64	53.57	13.79	22.26	10.76
	Low	7	16.43	4.76	23.57	6.90	8.68	3.58
	Overall	27	39.81	18.47	50.74	21.47	21.53	12.92

Table 1 displays the average N-gain in students' MR enhancement, categorized by high, medium, low, and overall PMA for both experimental and control groups. The average N-gain score of students' MR increases by 52.32 when the GPBL model is utilized, surpassing the CL model's score of 21.53. The mean enhancement in MR N-gain score for high PMA students utilizing the GPBL model is 78.32, surpassing the CL model's score of 34.82. The mean enhancement in MR N-gain score for medium PMA students utilizing the GPBL model is 53.62, surpassing the CL model's score of 22.26. The mean enhancement in MR N-gain score for low PMA students utilizing the GPBL model is 23.93, surpassing the CL model's score of 8.68.

Table 2 shows the normality test results for the overall mathematical reasoning N-gain scores and students' PMA.

Table 2. Normality Test Results of Mathematical Reasoning N-gain Score

PMA	Learning Group	N	Kolmogorov-Smirnov			Shapiro-Wilk			H_0
			Statistic	df	Sig.	Statistic	df	Sig.	
High	Experiment	7	0.251	7	0.200	0.822	7	0.068	Accepted
	Control	6	0.227	6	0.200	0.896	6	0.350	Accepted
Medium	Experiment	13	0.177	13	0.200	0.900	13	0.135	Accepted
	Control	14	0.155	14	0.200	0.952	14	0.592	Accepted
Low	Experiment	7	0.186	7	0.200	0.903	7	0.352	Accepted
	Control	7	0.322	7	0.027	0.704	7	0.004	Rejected
Overall	Experiment	27	0.146	27	0.148	0.929	27	0.067	Accepted
	Control	27	0.128	27	0.200	0.934	27	0.086	Accepted

Table 2 presents the MR N-gain scores of students categorized by high, medium, low, and overall PMA, derived from the normality tests of both the experimental and control groups. The normality test results for the MR N-gain of students in the entire experimental group utilizing the GPBL model yielded a significance value. The value of 0.067 exceeds 0.05, signifying that the control group utilizing the CL model achieved statistical significance. 0.086 is more significant than 0.05. The MR N-gain scores of students subjected to the GPBL and CL models exhibit a normal distribution. The normality test results for the MR N-gain of high PMA students in the experimental group, subjected to the GPBL model, revealed a significant value. 0.068 exceeds 0.05, while the control group utilizing the CL showed significance. 0.350 is more significant than 0.05. The N-gain score of MR for high PMA students subjected to the GPBL and CL models demonstrates a normal distribution. The normality test results for the MR N-gain of the medium PMA experimental group treated with the GPBL model revealed a significant value. 0.135 surpasses 0.05, suggesting that the control group utilizing the CL model achieved statistical significance. 0.592 is more significant than 0.05. The N-gain score of MR for medium PMA students subjected to the GPBL and CL models demonstrates a normal distribution. The normality test results for the MR N-gain of low PMA students in the experimental group using the GPBL model yielded a significant value. 0.352 exceeds 0.05, indicating that the control group utilizing the CL model exhibited statistical significance. 0.004 is less than 0.05. The enhancement in N-gain of MR among low PMA students subjected to the GPBL model follows a normal distribution. The CL model fails to exhibit a normal distribution.

Table 3 shows the homogeneity test results for the overall mathematical reasoning N-gain scores and students' PMA.

Table 3. Homogeneity Test Results of Mathematical Reasoning N-gain Score

PMA	Lavene Statistic	df1	df2	Sig.	H_0
High	3.304	1	11	0.096	Accepted
Medium	5.743	1	25	0.024	Rejected
Low	-	-	-	-	-
Overall	16.745	1	52	0.001	Rejected

Table 3 presents the homogeneity test results for students' MR N-gain scores, categorized by high, medium, low, and overall PMA from the experimental and control groups. Homogeneity tests performed on the N-gain scores of students' MR in both the experimental and control groups produced significant results. 0.001 is less than 0.05. The analysis reveals substantial differences in the aggregate N-gain scores of students' MR when evaluated under the GPBL model compared to the CL model. The N-gain scores of MR in the experimental group and the high PMA control group are statistically significant based on the homogeneity test results. 0.096 is greater than 0.05. This indicates that the MR N-gain scores of high PMA students subjected to the GPBL and CL models exhibit comparable variance. The MR N-gain values in both experimental and control groups exhibiting mild PMA demonstrate significant uniformity. 0.024 is less than 0.05. The MR N-gain scores of medium PMA students exposed to the GPBL and CL models exhibit notable differences. Neither the experimental nor the control groups conducted the N-gain score homogeneity test, as the low PMA experimental group showed a normal distribution, whereas the control group did not.

Table 4 shows the test results for the difference between the two average N-gain scores of overall mathematical reasoning and students' PMA.

Table 4. Results of Two Mean Difference Test of Mathematical Reasoning N-gain Score

PMA	Test Statistic	Sig. (2-tailed)	H_0	Mean Diff. Test Conc.
High	t-test	0.001	Rejected	There is a Significant Differences
Medium	t'-test	0.001	Rejected	There is a Significant Differences
Low	Mann-Whitney test	0.002	Rejected	There is a Significant Differences
Overall	t'-test	0.001	Rejected	There is a Significant Differences

Table 4 presents the difference test results comparing MR N-gain scores among high, medium, low, and overall PMA categories. The test results indicate a significant difference between students' mean MR N-gain scores combined in the experimental and control groups. The p-value of 0.001 (two-tailed) is below the significance threshold of 0.05. The enhancement in MR among students utilizing the GPBL model was significantly superior compared to those employing the CL model. The difference test results indicate that the means of students' MR N-gain scores in the experimental and control groups with high PMA are statistically significant. The p-value of 0.001 (two-tailed) is below the significance threshold of 0.05. The enhancement in MR for students with elevated PMA utilizing the GPBL model was significantly superior compared to those employing the CL model. The disparity in test results between the average N-gain scores of students' MR in the experimental group and the control group with medium PMA is statistically significant. The p-value of 0.001 (two-tailed) is below the significance threshold of 0.05. The enhancement in MR for students with average PMA utilizing the GPBL model was significantly superior to that of students employing the CL model. The difference test results indicate that the average N-gain scores of students' MR in the experimental and control groups with low PMA are statistically significant. The p-value of 0.002 (two-tailed) is less than the significance threshold of 0.05. The enhancement in MR for students with low PMA employing the GPBL model was significantly superior to that of students utilizing the CL model.

Additionally, researchers conducted interviews to identify the factors that hinder students' capacity to address MR problems associated with three-dimensional geometry.

Researcher : *"What challenges did you encounter in addressing the three-dimensional topic?"*

Student : *"I faced difficulties in visualizing the problem in three dimensions. In formulating the problem, I encountered difficulties identifying the specific issue."*

Researcher : *"What is your perspective on the application of GeoGebra?"*

Student : *"GeoGebra aids in the construction and visualization of three-dimensional problems, allowing for an accurate perception of three-dimensional shapes from various perspectives, thus enhancing problem-solving capabilities."*

Figures 4 and 5 show the results of students' constructions in solving mathematical reasoning problems on three-dimensional geometry using GeoGebra.

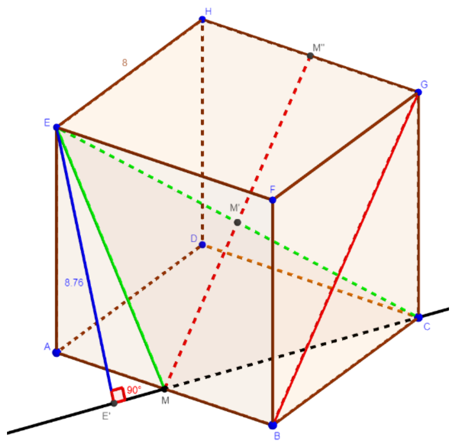

Figure 4. Distance from Point E to Line MC

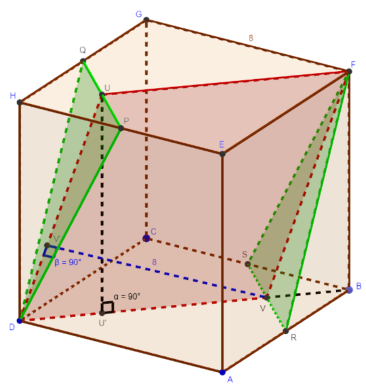

Figure 5. Distance from Plane SFR to Plane QDP

The GeoGebra-assisted Problem-Based Learning (GPBL) model has markedly improved students' mathematical reasoning, particularly in geometry and other mathematical subjects necessitating visual comprehension (Septian et al., 2020). GeoGebra is a technology-based learning aid that enables students to dynamically investigate and manipulate mathematical objects. This allows students to see and comprehend mathematical ideas more tangibly and graphically, improving their grasp of the interrelations among mathematical entities in three-dimensional geometry (Gökçe & Güner, 2022). Using GeoGebra, students may do direct experiments and graphically represent and understand mathematical issues, enhancing their reasoning abilities (Awaji, 2021).

The GPBL syntax demands that students participate in rigorous problem-solving, confronting real-world scenarios that require the application of mathematical principles to derive answers. GeoGebra is a tool for visualizing and modeling mathematical scenarios in this study (Arnawa & Fitriani, 2022). Students must formulate hypotheses, validate them via experimentation, and assess the outcomes, fostering critical and logical thinking (Hanifah et al., 2023). In three-dimensional geometry, students may use GeoGebra to manipulate objects like cubes, pyramids, or prisms and see how dimension alterations influence geometric features (Widada et al., 2021). This fortifies their comprehension and augments their capabilities in spatial and mathematical reasoning. GeoGebra effectively visualizes geometry and enhances geometric and digital abilities (Schmid et al., 2023).

The foundational principle of GPBL is social constructivism, which highlights learning as an active and collaborative endeavor (Birgin & Topuz, 2021). Within the framework of GPBL, learning transpires via students' engagement with pertinent concerns and ancillary technologies. The process of knowledge production emphasizes how students develop their comprehension via direct experience and reflection on faced challenges (Santos-Trigo, 2019). As an educational tool, GeoGebra facilitates flexible and exploratory learning for students (Condori et al., 2020). Students can enhance their mathematical reasoning by linking theory with practical application. This model aids students in developing mathematical reasoning, which is crucial in mathematics. Numerous studies corroborate the efficacy of the GPBL model in improving students' mathematical reasoning. Kustiawati and Siregar (2022) have shown that problem-solving using GeoGebra is significantly associated with mathematical reasoning in geometry education. Geometry is a primary domain for studying evidence and argumentation. It is defined by the interplay between the visual representation of geometric features and the intellectual comprehension of their significance essential for formulating precise explanations (Ramírez-Uclés & Ruiz-Hidalgo, 2022). Pérez and Ortega (2019) demonstrated that problem-based learning, facilitated by GeoGebra software, enhances the educational process, enabling students to connect geometric concepts to real-life applications. Moreover, Muchlis et al. (2023) discovered that the GeoGebra program facilitates problem-solving and idea exploration using inductive reasoning in plane and spatial geometry. Dynamic software facilitates students' comprehension of ideas and the resolution of three-dimensional issues such as distance, angles, and cross-sections. Consequently, this application may be used to demonstrate right triangles while instructing on the ideas of distance and angles, aiding in calculating distances and angle measurements. The Pythagorean theorem, the area of a triangle, and trigonometry are interconnected (Nurjanah et al., 2020).

CONCLUSION

The initial phase of the GPBL model improved students' mathematical reasoning related to three-dimensional concepts by guiding them toward the spatial geometry problem. Students synthesize their prior knowledge with the current subject matter. Students examine mathematical problems through the application of pertinent mathematical principles. Students construct the three-dimensional problem and subsequently solve it using algebraic procedural fluency. Students evaluate and analyze the results of problem-solving using GeoGebra. The three-dimensional problem is constructed and demonstrated using GeoGebra. Students conclude by solving three-dimensional problems by applying logical principles derived from GeoGebra. Geometry study improves students' mathematical reasoning abilities.

This research has several implications that can affect learning practices. This research offers significant insights for educators in geometry learning, advocating for implementing the Problem-Based Learning (PBL) model enhanced by technology like GeoGebra. The PBL model enables students to cultivate problem-solving abilities by comprehensively examining geometric concepts. This research can improve students' comprehension of three-dimensional geometry by analyzing their thought processes and problem-solving approaches in geometric contexts. This offers essential insights for developing more effective educational strategies for students with varying foundational mathematical abilities. Third, technology in geometry education, exemplified by tools like GeoGebra, has demonstrated efficacy in visualizing abstract geometric concepts, thereby enhancing the interactivity and engagement of the learning experience for students. This underscores the significance of incorporating technology into geometry education. The GeoGebra-assisted PBL model promotes the development of mathematical reasoning in students and enhances their ability to identify and solve complex mathematical problems, thereby contributing to the advancement of their cognitive abilities.

This study presents several limitations that warrant consideration. This study is limited by a small sample size, involving only a few schools or students, which may hinder the generalizability of the findings to a broader student population. This research is confined to particular three-dimensional geometry topics, potentially impacting students' abilities and understanding transfer to other mathematical areas not addressed in this study. This research was conducted over six meetings, which may be inadequate for evaluating the long-term effects of GeoGebra on geometry learning.

Future research should enhance these findings by incorporating a more extensive and diverse sample, encompassing different types of schools and geographical regions, to assess the results' generalizability. Applying GeoGebra to teach various geometry or mathematics topics can yield additional insights into its effectiveness in improving mathematical reasoning among high school students. Subsequent research may investigate variables, including students' learning styles, motivation and interest in learning, and social support, along with additional internal and external factors that could affect the learning process. Moreover, further research can create and evaluate diverse GeoGebra-based learning tools and modules designed to meet students' needs in understanding more intricate geometric concepts. Qualitative research can be conducted to explore students' cognitive processes in solving geometric problems and to investigate the challenges they encounter when utilizing GeoGebra for geometry. Further research is anticipated to enhance mathematics teaching practices, especially regarding geometry instruction in high schools.

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