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PROTON DRIPLINE FOR ISOTONE N = 18, 20, AND 22 USING MODIFIED RELATIVISTIC MEAN FIELD (MRMF) MODEL

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ABSTRACT

Determining the position of one- and two-proton dripline for isotone of N = 18, 20, and 22 has been studied through Modified Relativistic Mean Field (MRMF). The model exemplifies three impacts, namely isovector-isoscalar coupling, tensors, and electromagnetic exchange through five parameter set variations. The position of one- and two-proton dripline for the isotones is predicted by applying two methods, which are two-proton separation energy, and Fermi energy. The research shows that the prediction of one- and two-proton dripline for isotone of N = 18, and N = 20 is positioned at Z = 22 and Z = 26 consecutively. Then, the prediction of one- and two-proton dripline for isotone of N = 22 has two positions, Z = 26 and Z = 28. The calculation result indicates that the position prediction for isotone of N = 18, N = 20, and N = 22 is following the research result conducted by Nazarewicz with RMF+NLSH model [1]. Meanwhile, isovector-isoscalar coupling, tensors, and electromagnetic exchange do not affect massively for the position prediction of two-proton dripline. However, the three methods affect one-proton dripline.

Keywords: proton dripline, Modified Relativistic Mean Field (MRMF)

INTRODUCTION

Nuclei consist of protons and neutrons with a variety of amount combination. The nuclear chart divides into several regions, such as the valley of β-stability (stable nuclei, non-radioactive or long-lived) and terra incognita region (radioactive nuclei with an extreme ratio of neutron and proton) [2]. Terra incognita region corresponds to the driplines as predicted by the mass model of Tachibana, Uno, Yamada, and Yamada (TUYY) [3]. Proton dripline area, which is out of stability line, has a ratio of proton and neutron by 2.0 – 2.5, and the condition is also applied for neutron dripline area. Nuclei near the proton or neutron driplines do not have a uniform distribution. Moreover, the forces between protons and neutrons are no longer strong enough to hold them together [4].

Proton dripline is fascinated to examine further because of the repulsive Coulomb force, which gains strength as more protons are added. Proton dripline has been studied since the beginning of nuclear reactions because it is much closer to the valley of stability than the neutron dripline. Besides, fusion-evaporation reactions are always populated proton-rich nuclei making the proton dripline more accessible than the neutron dripline [5]. The proton dripline is positioned at isotone chains, while the neutron dripline is positioned at isotope chains.

Modified Relativistic Mean Field (MRMF) model is a standard Relativistic Mean Field (RMF) model were augmented with isovector-isoscalar coupling, tensors, and electromagnetic exchange terms. The relativistic model describes the nucleus as a system of Dirac nucleons interacting via meson fields. Relativistic Mean Field (RMF) model yields acceptable spin-orbit splitting and suitable for high-density extrapolation. In term of nuclear symmetry case, the density of saturation comes up in the calculation result without inserting three object potentials. The condition cannot be realized if using non-relativistic model, as an example, Skyrme Hartree-Fock. Thus, the MRMF model is sufficiently reliable to investigate the proton dripline, such as non-relativistic model [4].

This research uses light-nuclei having proton dripline, which are isotone of $N = 18$, $N = 20$, $N = 22$. The position of one- and two-proton dripline for the isotones are investigated further by applying MRMF Model with five parameter set variations. The five variations show the correlation between position prediction for one- and two-proton dripline and three impacts, that is isovector-isoscalar coupling, electromagnetic exchange, and tensors.

RESEARCH METHOD

The method is a theoretical calculation consisting of analytic and numeric computation. The numeric computation applies for Fortran Program, while the analytic calculation gains Dirac Nucleon equation, the motion equation of meson σ, meson δ, meson ω, and meson ρ, Lagrangian Density, as well as Hamiltonian Density with MRMF Model. Then, the equations are inserted into Fortran Program to gain parameter set, binding energy, and other particle properties. Here in after, the result is compared with the experimental result (Magic Number Particle), as shown in FIGURE 1.

Modified Relativistic Mean Field (MRMF) Model predicts the symmetry energy is more suitable with the experimental result than RMF standard model. The values of symmetry energy are in the range of 25 – 35 MeV [6]. The symmetry energy is required to change a proton into a neutron. The contribution of energy symmetry cannot be applied if they have a similar amount. However, if the amount of proton and neutron is different, the contribution of symmetry energy and binding energy will be more massive. In conclusion, symmetric energy of model classifies potentially whether a parameter is worthy or not to be applied in research.

MRMF model is a standard Relativistic Mean Field (RMF) model were augmented with isovector-isoscalar coupling, tensors, and electromagnetic exchange terms. Thus, the Lagrangian Density represents the equation of motion, and the Hamiltonian Density represents the equation of energy. The explicit form of the Lagrangian Density of MRMF model is

$$\mathrm{L}_{MRMF} = \mathrm{L}_N + \mathrm{L}_M + \mathrm{L}_{Lin} + \mathrm{L}_{Nl} + \mathrm{L}_T + \mathrm{L}^4_{\omega\rho} + \mathrm{L}^{EM}_{exc} \tag{1}$$

$$\begin{aligned}
\mathrm{L}_N &= \sum_{j=1}^{A} \bar{\psi}_j \left(i\gamma^\mu \partial_\mu - M \right) \psi_j \\
\mathrm{L}_M &= \frac{1}{2}\left(\partial_\mu \phi \partial^\mu \phi - m_\sigma^2 \phi^2 \right) - \frac{1}{2}\left(\frac{1}{2} V_{\mu\upsilon} V^{\mu\upsilon} - m_\omega^2 V_\mu V^\mu \right) - \frac{1}{2}\left(\frac{1}{2} R_{\mu\upsilon} R^{\mu\upsilon} - m_\rho^2 R_\mu R^\mu \right) \\
&+ \frac{1}{2}\left(\partial_\mu \vec{d} \partial^\mu \vec{d} - m_\delta^2 \vec{d}^{\,2} \right) - \frac{1}{2} \partial^\upsilon A^\mu \partial_\upsilon A_\mu \\
\mathrm{L}_{Lin} &= \sum_{j=1}^{A} \bar{\psi}_j \left(g_\sigma \phi - g_\omega V_\mu \gamma^\mu + \frac{1}{2} g_\rho R_\mu \gamma^\mu \tau + g_\delta \vec{d} \tau - eA^\mu \frac{1+\tau_0}{2} \gamma^\mu \right) \psi_j \\
\mathrm{L}_{Nl} &= -\left(\frac{k_3}{6M} g_\sigma m_\sigma^2 \phi^3 + \frac{k_4}{24M^2} g_\sigma^2 m_\sigma^2 \phi^4 \right) + \frac{1}{24} \xi_0 g_\omega^2 \left(V_\mu V^\mu \right)^2 \\
\mathrm{L}_T &= -\sum_{j=1}^{A} \bar{\psi}_j \left(\frac{f_\omega}{2M} \partial^\upsilon V^\mu i\gamma_\mu \gamma_\upsilon + \frac{f_\rho}{4M} \partial^\upsilon R^\mu \tau i \gamma_\mu \gamma_\upsilon \right) \psi_j \\
\mathrm{L}^4_{\omega\rho} &= \frac{\eta_{2\rho}}{4M^2} g_\omega^2 m_\rho^2 V_\mu V^\mu R_\mu R^\mu \\
\mathrm{L}^{EM}_{exc} &= C^{EM}_{exc} \left[\frac{3}{4} e^2 \left(\frac{3}{\pi} \right)^2 \right] \rho_p^{4/3} \left[1 - \frac{2}{3M^2} \left(3\pi^2 \right)^{2/3} \rho_p^{2/3} \right]
\end{aligned} \tag{2}$$

MRMF model consists of seven-terms, those are nucleon, meson field (meson σ, meson ω, meson ρ, and meson δ), a linear term, non-linear term, tensor meson ω, and meson ρ, isovector-isoscalar coupling, and electromagnetic exchange term. The interaction of meson field consists of four mesons, namely meson σ (meson scalar isoscalar and attractive), meson δ (meson scalar isovector having a role in symmetry energy), meson ω (meson vector isoscalar which is repulsive), and meson ρ (meson vector isovector which depends on the effect of isospin). Meson δ and meson ρ have a role prominently in symmetry energy. If proton and neutron have a similar amount, the contribution of meson δ and meson ρ could be minimum. On the other hand, if the nuclei only consist of the neutron, the contribution of meson δ and meson ρ will be maximum [4].

The linear term is a self-coupling term for each meson, namely meson σ, meson ω, meson ρ, and meson δ. Non-linear terms are the higher order self-coupling terms for meson σ and meson ω. One of the significant effects produced by this terms is softening the nuclear incompressibility (meson σ) [8], and it yields optimum behavior for the EOS (equation of state) and neutron star matter at high densities (meson ω). Tensor terms consist of tensor isoscalar and isovector. Both tensors play a crucial role in bulk properties finite nuclei and single particle spectra. The cross-coupling isoscalar isovector term is adjusted to yields reasonable agreement with the energy per neutron for the dilute neutron matter [7]. This interaction potentially converts the density dependence on the coefficient of symmetry energy and the neutron skin thickness in the heavy nuclei without affecting the properties of finite nuclei.

Electromagnetic exchange term is the best-known part of the nuclear Hamiltonian, and both its direct (Hartree) and exchange (Fock) terms can be exactly calculated at the mean-field level. In this term, we used the Local Density Approximation (LDA) form. The effects which are missing by performing LDA approximation is assumed to be effectively absorbed by free parameter C_{exc}^{EM} [8].

TABLE 1. Variations of the Parameter Set for MRMF Model

Parameter Set	Tensors	Electromagnetic Exchange	Isovector-Isoscalar Coupling
P0	–	–	0.0
PT00	√	–	0.0
PTE00	√	√	0.0
PT40	√	–	0.40
PTE40	√	√	0.40

The MRMF model consists of five variations of the parameter set. The variations of parameter sets are applied to identify an effect of terms of isovector-isoscalar coupling, tensors, and electromagnetic exchange to the position prediction of one- and two-proton dripline (See TABLE 1).

The equation of motion, which is supported by the Hamiltonian equation, is applied to find out an energy density equation. Then, the equation of energy density is applied to predict the position of one and two-proton dripline. The prediction of one-proton dripline uses Fermi energy, where the Fermi energy of proton will be close to zero ($\epsilon_f \sim 0$) if the core is in a dripline form [9]. Fermi energy is the bottom level of energy at the condition that the atomic nucleus is still able to hold its nucleus (the nucleus is at sta able level). If Fermi energy is below zero ($\epsilon_f < 0$), the nucleus can be hold. However, if Fermi energy is above zero ($\epsilon_f > 0$), the nucleus will exuviate (it is at an unstable level).

The prediction of the position of two-proton dripline can be determined using the separation energy of two protons. Two-proton dripline occur when $S_{2p} \sim 0$ [10][11]. Separation energy of two proton is a energy is needed to release two protons from a nucleus. Separation energy are calculated from the binding energy difference between two adjacent nuclei [12]. Separation energy of two protons are written as follows [11]:

$$S_{2p} = E_B(Z,N) - E_B(Z-2,N) \tag{3}$$

Where, $E_B(Z,N)$ is binding energy of the nuclei with the number of proton Z and the number of neutron N.

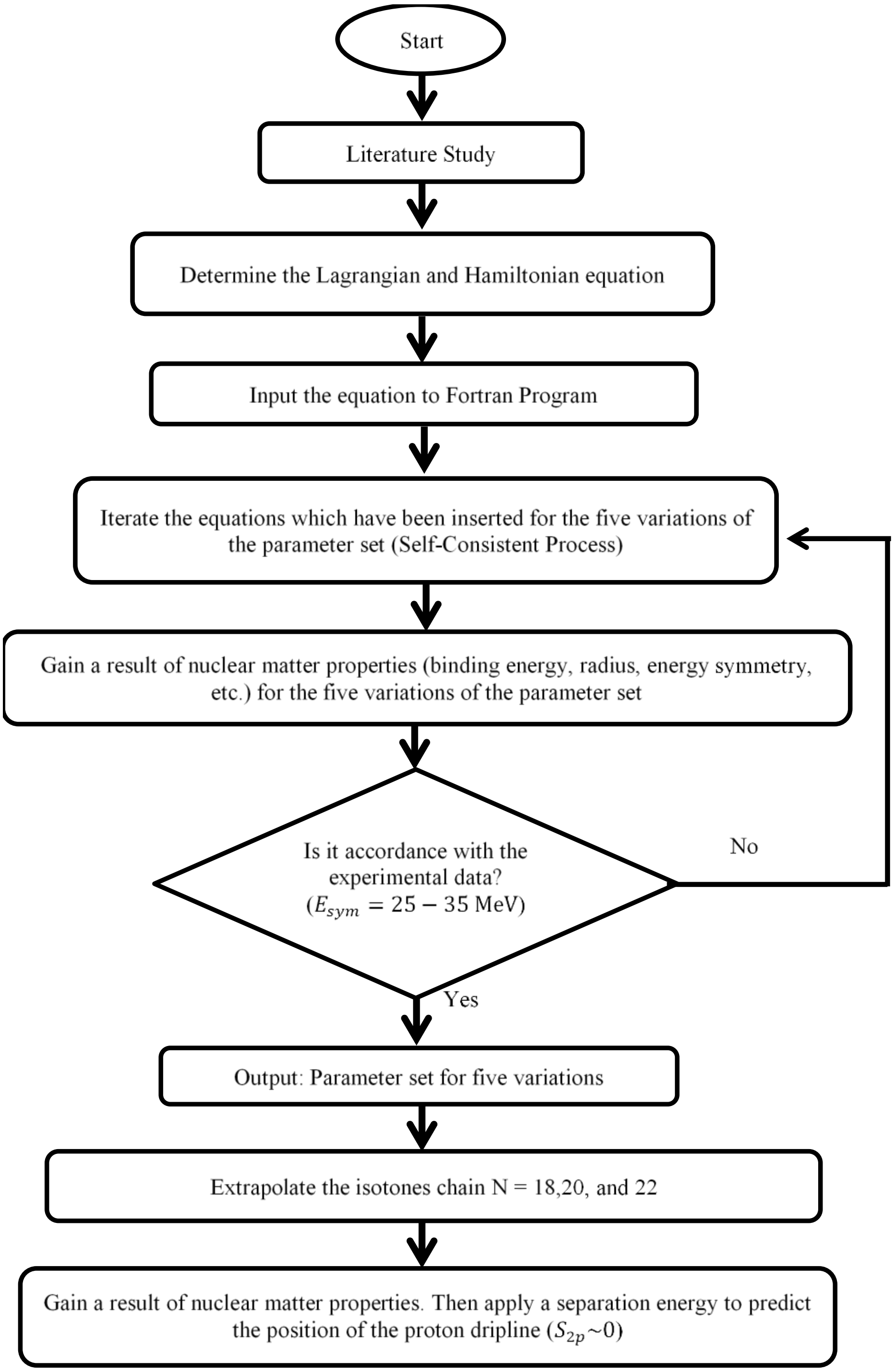

FIGURE 1. Flowchart Research

RESULTS AND DISCUSSION

Based on this research, the prediction of proton dripline is positioned at isotone chain of N = 18, N = 20 and N = 22. It is applied with Fermi energy, while the position prediction of two-proton dripline is applied with separation energy. The result of prediction of one- and two-proton dripline in N = 18 isotones are presented in FIGURE 2. The prediction result of one-proton dripline for isotone of N = 18 is various. Position prediction of one-proton dripline by applying parameter set of P0 and PT40 is at Z = 20.

Meanwhile, position prediction of one-proton dripline by applying parameter set of PT00, PTE00, and PTE40 as well as applying experimental result is at Z = 22 and Z = 24 respectively. In FIGURE 1, the graphic shows that tensors and electromagnetic exchange affect significantly on the result of predictions. Contribution of tensors for the position prediction is reflected by parameter set of P0 and PT00, while contribution of electromagnetic exchange is reflected by parameter set of PT40 and PTE40. Tensors and electromagnetic exchange assist the nuclei to be steady leady not to rapidly decreasing Fermi Energy. The relative error between the calculation of five variations of the parameter set with the experimental results shows that PTE00 and P0 parameter sets have the smallest relative errors. Isovector-isoscalar coupling affects the relative error which can be seen at Z = 24. The higher value of the isovector-isoscalar coupling leads to higher relative error.

The prediction of two-proton dripline for N = 18 isotone is positioned at Z = 22 by applying all variations of the parameter sets, experimental results (AME 2003) [13], and the results of research by Nazarewicz, *et al*. with RMF+NLSH model [1]. Based on the results of this analysis, in other words, the position of one- and two- proton dripline for isotone of N = 18 are at Z = 22. On the other hand, this prediction cannot reflect the effect of tensors, electromagnetic exchange, and isovector-isoscalar coupling. The smallest relative error for PTE40 parameter set which the RMF+NLSH model follow. The position prediction indicates that the results of the calculation are appropriate with the RMF+NLSH model. Tensors and electromagnetic exchange terms reduce the relative error in the separation energy of two protons. The isovector-isoscalar coupling will have a good effect if it collaborates with tensors and electromagnetic exchanges. However, if the isovector-isoscalar coupling collaborates with tensors, it will be diluted the separation energy of two protons.

Determining the prediction of one-proton dripline on N = 20 isotone is by applying Fermi Energy, and this condition is also applied for isotone of N = 18 (FIGURE 3). The prediction of the position of one-proton dripline is at Z = 24 by applying parameter set of P0 and PT40, while at Z = 26 by applying parameter set of PT00, PTE00, PTE40 parameter sets, as well as experimental results. The prediction result reflects the effect of electromagnetic exchange to isotone of N = 20 following to isotone of N = 18. The relative error between the calculation results and the experimental result shows that PTE00 parameter set has the smallest relative error. The existence of tensors and electromagnetic exchange makes the error relatively smaller. However, the isovector-isoscalar coupling has the opposite value of errors.

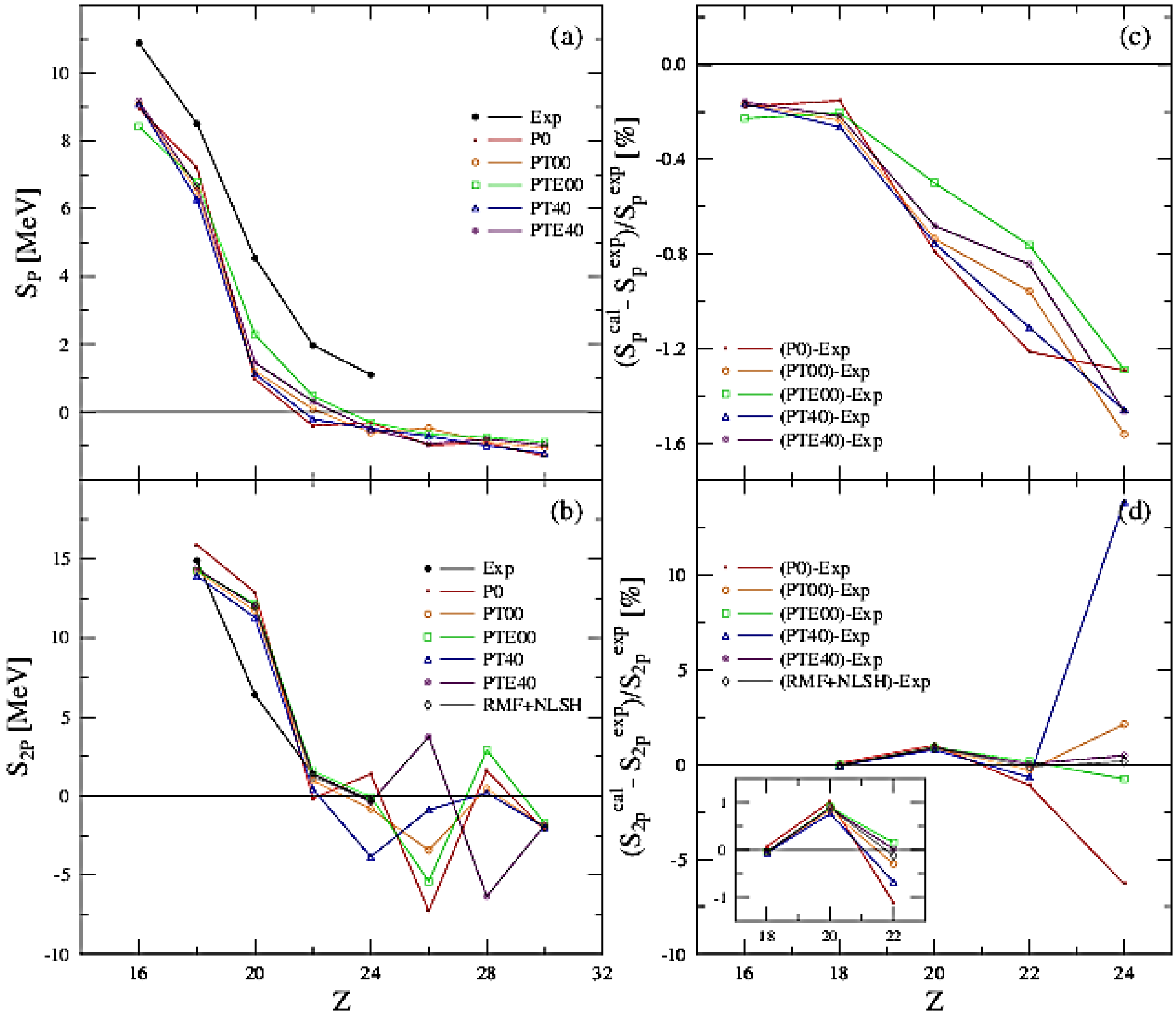

FIGURE 2. Fermi energy (a) and the separation energy of two protons (b) for five variations of the parameter set, experimental results (AME2003), and the result of research by Nazarewicz *et al.* for N = 18 isotone. The percentage of relative errors between the calculation of S_p and S_{2p} with the experimental results on the prediction of one-proton dripline (c) and two-protons dripline (d).

Prediction of two-proton dripline applying the separation energy of two protons on isotone of N = 20 shows that the position prediction of two-proton dripline will be at Z = 24 if parameter set of P0 is applied, while if the other four parameter set, experimental results, and RMF+NLSH models are applied, the position of two-proton dripline will be predicted at Z = 26. The result indicates that tensors, electromagnetic exchange, and isovector-isoscalar coupling do not have a significant effect of these predictions, as well as the percentage of relative errors. The result of the research using the RMF+NLSH model is also similar to the four parameters set variations. In conclusion, the position of one- and two- proton dripline on isotone of N = 20 is at Z = 26.

The prediction of position one- and two- proton dripline on isotone of N = 22 is presented in FIGURE 4. Applying Fermi energy, five variations of parameter set have similar position prediction for one-proton dripline which is at Z = 26. However, the position prediction for one-proton dripline with the experimental result is at Z = 28. The position prediction of a proton dripline shows that tensors, electromagnetic exchange, and isovector-isoscalar coupling do not affect the results. However, the percentage of relative errors shows a significant effect. Tensors and electromagnetic exchanges lead the relative error to be smaller. It is along with the prediction result for isotone N = 20.

Prediction of the position of two-proton dripline applying separation energy also shows the same results for the five variations of parameter sets, experimental results, and the RMF+NLSH model, which is at Z = 28. Having different condition with one-proton dripline, the percentage of relative errors for two-proton dripline indicates otherwise. The existence of tensors and electromagnetic exchange lead the error relatively larger. For isotone of N = 22, it also has difference position predictions of one- and two- proton dripline. One-proton dripline is positioned at Z = 26, while two-proton dripline at Z = 28.

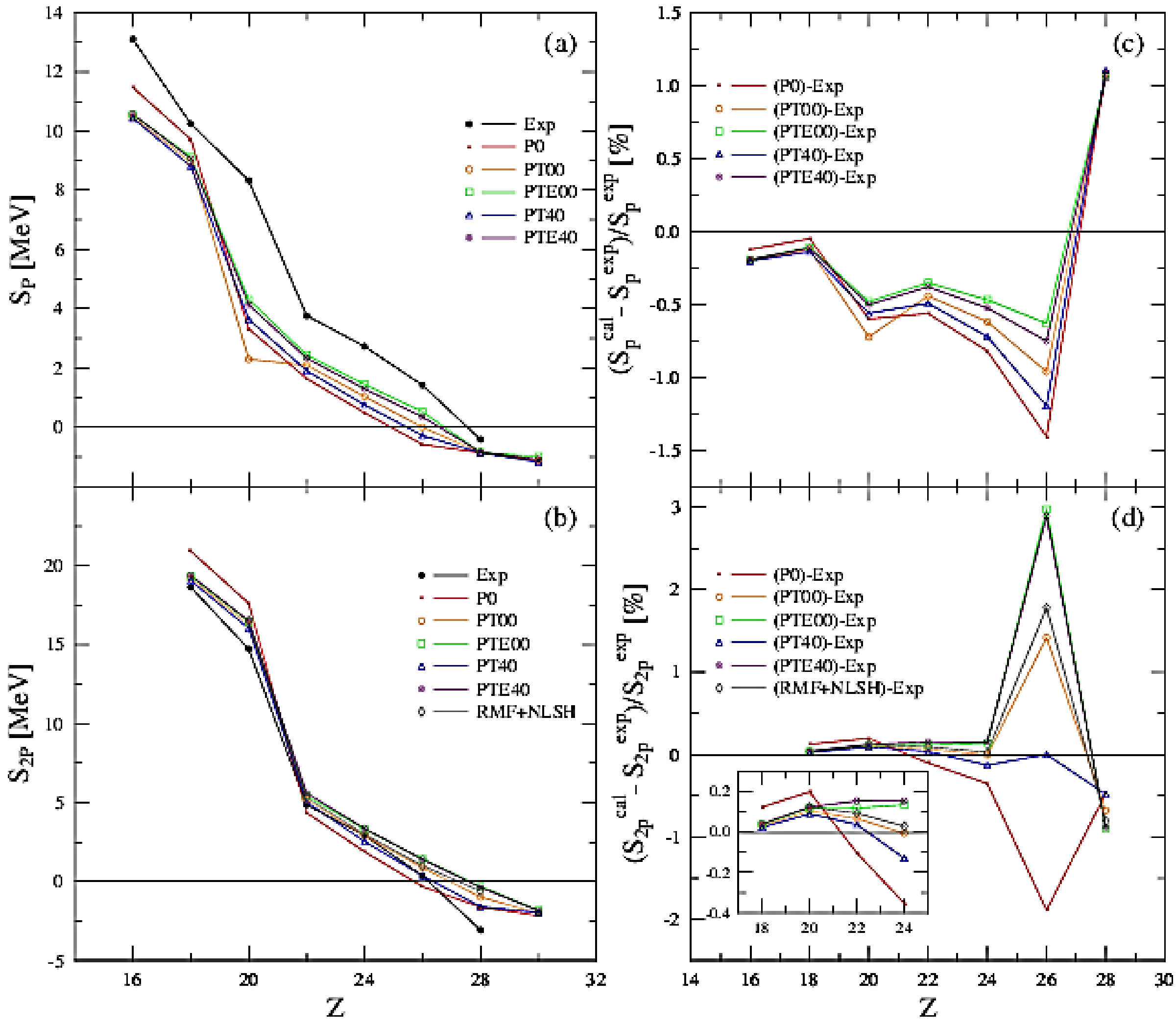

FIGURE 3. Fermi energy (a) and the separation energy of two protons (b) for five variations of the parameter set, experimental results (AME2003), and the result of research by Nazarewicz *et al*. for N = 20 isotones. The percentage of relative errors between the calculation of S_p and S_{2p} with the experimental results on the prediction of one-proton dripline (c) and two-protons dripline (d).

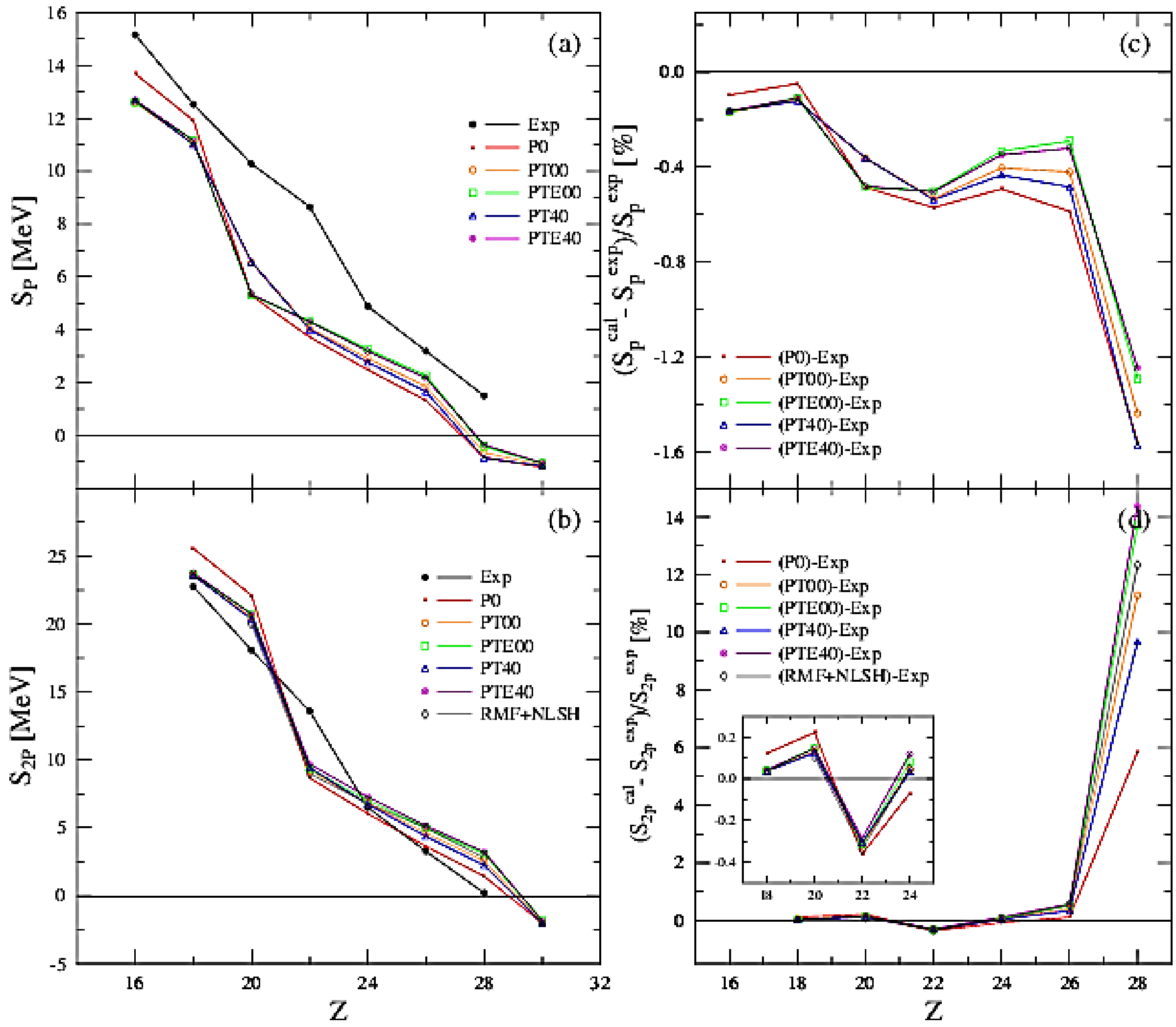

FIGURE 4. Fermi energy (a) and the separation energy of two-protons (b) for five variations of the parameter set, experimental results (AME2003), and the result of research by Nazarewicz *et al*. for N = 22 isotones. The percentage of relative errors between the calculation of S_p and S_{2p} with the experimental results on the prediction of one-proton dripline (c) and two-protons dripline (d).

SUMMARY

The position of one- and two- proton dripline for isotone N = 18 and N = 20 is at Z = 22 and at Z = 26 consecutively. However, on isotone N = 22, one-proton dripline is positioned at Z = 26, and two-proton dripline is at Z=28. Tensors, the electromagnetic exchange term has a significant effect on the results of predictions of one- and two-proton dripline for isotone of N = 18 and N = 20. Whereas, the isovector-isoscalar coupling affects the percentage of relative error. The results of prediction of two-proton dripline for isotone of N = 18, N = 20 and N = 22 are consistently by the results of a research conducted by Nazarewicz with the RMF+NLSH model.

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