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Hydrostatic Mass of Galaxy Clusters within Eddington-inspired Born Infeld Theory Modified by Generalized Uncertainty Principle

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ABSTRACT

The mass of galaxy cluster systems can be determined by calculating the hydrostatic equation for such systems. In this study, we derive the hydrostatic mass of galaxy cluster systems within the Eddington-inspired Born-Infeld (EiBI) theory, a modified theory of gravity. The EiBI theory is further modified by incorporating the generalized uncertainty principle (GUP) into its formulation. The GUP affects the mathematical expression of the temperature in galaxy clusters, leading to modifications in the clusters' equation of state (EoS), which is also an essential mathematical tool in hydrostatic equation calculations. This incorporation is motivated by the need to explore quantum gravitational effects on cosmological scales, bridging a fundamental gap between a modified theory of gravity and quantum mechanics. This work is significant in that it introduces the effect of the GUP, implemented through a modification of the temperature, within the framework of EiBI gravity. Using the derived formulation, we calculate the mass of 12 galaxy clusters and compare the results with observational data. The calculations reveal a significant reduction in the masses of these galaxy clusters to the order of 10^{-19} $M_{\odot}$. A result which is profoundly inconsistent with observational data, thereby challenging the physical viability of this specific EiBI-GUP framework for modelling large-scale structures like galaxy clusters.

Keywords: EiBI theory, galaxy clusters, GUP, hydrostatic mass

INTRODUCTION

Galaxy clusters are the largest virialized systems on an astronomical scale in the universe [1-8]. As massive structures, galaxy clusters are excellent natural laboratories for obtaining physical constraints on neutrino mass, graviton mass, and primordial non-Gaussianity limits [9]. Generally, the mass of galaxy clusters can be determined using X-ray data profiles, the Sunyaev-Zel'dovich (SZ) effect, and the gravitational lensing phenomenon [10-12]. For relaxed galaxy clusters (i.e., systems that have returned to equilibrium after being perturbed), the cluster mass can be calculated using the Newtonian hydrostatic equilibrium equation for spherical symmetry [9]:

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2}, \tag{1}$$

where P is the pressure, r denotes the radial distance from the cluster centre, G is the universal gravitational constant, M is the cluster mass, and ρ is the cluster density.

Bambi made significant contributions to the understanding of galaxy cluster mass by exploring the impact of dark energy under the assumption that it behaves as a cosmological constant Λ on relatively small scales [10]. This led to the formulation of the effective mass of galaxy clusters as $M_{\text{eff}} = M_0 - \frac{8}{3}\pi r^3 \rho_\Lambda$, where M_0 is the Newtonian mass of the galaxy cluster, ρ_Λ is the energy density associated with the cosmological constant $\Lambda = 8\pi G \rho_\Lambda$. Utilizing Bambi's framework, Desai (2022) evaluated the masses of several galaxy clusters and concluded that the general relativistic corrections to hydrostatic mass estimates using the Kottler metric [10] are negligible.

Gupta and Desai calculated the masses of 12 types of galaxy clusters using the Tolman-Oppenheimer-Volkoff (TOV) equation [9]. It is important to note that galaxy clusters are characterized by specific density and temperature distributions, denoted as $\rho(r)$ and $T(r)$. These quantities were derived by Vikhlinin et al. (2006) (Vikhlinin et al., 2006). The TOV-based cluster masses obtained by Gupta and Desai [9] were compared with data obtained from Chandra X-ray observations, which are calculated using the Newtonian hydrostatic equilibrium equation. The comparison revealed a negligible difference between the two mass estimates, on the order of $\sim 10^{-5}$. It is worth noting that the use of the TOV equation in the context of galaxy clusters is unconventional, as it is typically applied to compact objects such as neutron stars[13-15], strange stars [16], and boson stars [17]. Recent studies on determining the mass of galaxy clusters are found in the literature through the Splashback Radius [18].

Apryandi and Pattersons calculated the mass of galaxy clusters within the Eddington-inspired Born-Infeld (EiBI) theory, one of the modified gravities. The consideration of modified gravities is crucial in studying various cosmic objects [19]. In 2010, Bañados and Ferreira [20] resurrected Eddington's proposal for the gravitational action in the presence of a cosmological constant [21] and extend it to include matter fields. Over the past several decades, substantial observational evidence has indicated that if gravity is accurately described by general relativity, then a significant portion of the mass in galaxies and galaxy clusters must exist in the form of dark matter. This invisible component is essential to explain these cosmic systems' observed dynamics and structural properties. In addition, dark energy presents another profound challenge that demands an explanation, as it is linked to the observed accelerated expansion of the Universe. Here, the modified gravities explain these problems [22]. Assuming the absence of dark matter and dark energy, Apryandi and Pattersons conducted a comparative analysis of the total mass of galaxy clusters predicted by the EiBI theory against the baryonic mass [19]. Within the framework of the EiBI theory, and using a parameter value of $\kappa = 5.80 \times 1040\ \mathrm{m}^2$, they determined the resulting slope to be 0.126 ± 0. This comparison highlights the implications of the EiBI theory on the distribution and estimation of mass in galaxy clusters. However, the results are not yet satisfactory, as the value needs to be close to unity to address the issues expected to be resolved by the EiBI theory.

On the other hand, by using the ideal gas equation of state (EoS), we can determine the hydrostatic mass $M(r)$, which is dependent on the temperature $T(r)$ and the density $\rho(r)$ [23-24]. The equation determining $M(r)$ has been widely used in numerous studies to determine various parameters, such as the total mass of galaxy clusters from X-ray data, the concentration-mass relationship, and mass estimates from observations. These parameters are utilized to study clusters in cosmological contexts or to investigate other fundamental physical parameters [25].

Interestingly, Belfaqih et al. demonstrated that the generalized uncertainty principle (GUP), an extension of the Heisenberg uncertainty principle (HUP), can significantly affect the gravitational properties of celestial objects [26]. Their study specifically explored the impact of GUP on the physical characteristics of white dwarfs. Then Oscar López-Aguayo et al. [27] and Giardino and Salzano [28] applied GUP in the context of beyond galaxy cluster, i.e. cosmology. Similarly, study by Apryandi et al. showed that GUP can alter the temperature formulation, thereby modifying the EoS [29]. While GUP effects are typically significant at the Planck scale, their manifestations can be observed in astrophysical systems through cumulative corrections to the equation of state and the thermal properties of matter.

In this work, we study the impact of GUP on the Newtonian equation and the EoS of the ideal gas by modifying the temperature formulation. Additionally, the influence of the EiBI theory on the Newtonian equation is investigated. It should be noted that in this study, two aspects can be modified: the EoS of the ideal gas and gravity. GUP can affect both, while the EiBI theory only affects gravity. This study performs GUP calculations to obtain temperature modifications, which indirectly modify the EoS. The modified EoS is applied to the EiBI theory to derive the formulation of the galaxy cluster mass. To the best of our knowledge, this is the first attempt to combine the EiBI gravity framework with the GUP in order to analyze the mass of galaxy clusters.

METHODS

This research was conducted using quantitative methods. This subsection contains the detailed equations used to derive analytical and numerical results. The numerical results were calculated based on the analytical calculations by substituting data from Gupta and Desai [9] and Vikhlinin et al. [30]. The numerical calculations were performed using the FORTRAN 77 programming language. The numerical calculations were performed using legacy FORTRAN

77 codes, which remain efficient for handling large-scale computations. Furthermore, we examine different choices of free parameters and found that the results do not exhibit significant variations. Therefore, in this work we adopted representative values, while leaving more refined implementations for future investigations.

We need to review the GUP to understand the derivation of analytical results. Initially, quantum mechanics predicted the relationship between the uncertainties of position x and momentum p through the HUP [31-33]

$$\Delta x \Delta p \geq \frac{\hbar}{2}. \tag{2}$$

Here, $\hbar$ denotes the Planck constant. This uncertainty principle is generalized into the GUP, which has the mathematical expression as [29]

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left[1 + \frac{\beta_0 l_p^2}{\hbar^2}(\Delta p)^2\right], \tag{3}$$

where β_0 denotes GUP's free parameter, and l_p is the Planck length. The value of the GUP parameter β_0 was selected based on bounds from contemporary literature [26], and the propagation of uncertainties from this parameter, as well as from the observed density profiles, was incorporated into the final mass estimates.

From the gravitational aspect, the hydrostatic equilibrium equation in the EiBI theory is required to obtain analytical results. Mathematically, this equation is given by [19]

$$\frac{dP}{dr} = -\frac{GM\rho}{r^2} - \frac{\kappa}{4} G\rho \frac{d\rho}{dr}, \tag{4}$$

where κ is the free parameter of EiBI theory.

RESULTS AND DISCUSSIONS

Analytical Results

The analytical calculations performed by formally deriving the mathematical formulation result in an expression for the galaxy cluster mass that is modified due to the presence of modified temperature T_{mod}, which is a consequence of the GUP. For the mathematical expression of T_{mod}, see EQUATION (24) in Apryandi et al. [29]. This mathematical form of T_{mod} needs to be simplified, resulting in

$$T_{\text{mod}} \approx \frac{2E}{k_B}\left[\frac{\pi\beta_0}{4}\ln\left(\frac{A}{4l_P^2}\right) - \frac{A}{l_P^2}\right], \tag{5}$$

where k_B is the Boltzmann constant, A is the area of a spherical surface enclosing a space with radius r. Note that the temperature has now been modified due to the presence of the GUP free parameter β_0. Remember that $E = Mc^2$, and at r_{500}, it holds that

$$M = \rho V_{500} = \rho \times \frac{4}{3}\pi(r_{500})^3. \tag{6}$$

According to Rahvar & Mashhon [34], ρ writes

$$\rho = \frac{\left(\frac{r}{r_c}\right)^{-\alpha}}{1+\left(\frac{r^2}{r_c^2}\right)^{3\beta-\frac{\alpha\prime}{2}}} \frac{n_0}{\left(1+\frac{r^\gamma}{r_s^\gamma}\right)^{\frac{\epsilon}{\gamma}}} + \frac{n_0'^2}{\left(1+\frac{r}{r_c'^2}\right)^{3\beta\prime}}, \tag{7}$$

where r is the radius of the galaxy cluster, r_c is the centre's radius ranging from tens to hundreds of kpc, r_s is the sphere radius around ~ 1000 kpc, n_0 is the density of the gas, n_0' is the density on the order of 10^{-1}; and α, α', β, β', and γ are constants from each galaxy profile. Here, ϵ is the limit of the physical form of the galaxy with a range of $\epsilon \leq 5$. Details of these entities' values can be referred to in Rahvar and Mashhoon [34].

So, EQUATION (5) can be solved to T_{mod} with relation $E = Mc^2$ then use EQUATION (6) and (7) for densitiy profile. Now T_{mod} reads

$$T_{\text{mod}} = \frac{8\pi r^3 c^2}{3k_B\left[\frac{\pi\beta_0}{4}\ln\left(\frac{A}{4l_P^2}\right) - \frac{A}{l_P^2}\right]} \frac{\left(\frac{r}{r_c}\right)^{-\alpha}}{1+\left(\frac{r^2}{r_c^2}\right)^{3\beta-\frac{\alpha'}{2}}} \frac{n_0}{\left(1+\frac{r^\gamma}{r_s^\gamma}\right)^{\frac{\epsilon}{\gamma}}} + \frac{n_0'^2}{\left(1+\frac{r}{r_c'^2}\right)^{3\beta'}}. \tag{8}$$

However, in the calculation of galaxy cluster's mass, we need the expression of $\frac{d(\ln T_{mod})}{d\,(\ln T_{mod})}$. Thus, we need to calculate $\ln T_{\text{mod}}$, i.e.

$$\ln T_{\text{mod}} = \ln 8\pi + 3\ln r + \ln c^2 + \ln\rho - \left[\ln 3k_B + \ln\left\{\frac{\pi\beta_0}{4}\left(\ln\frac{\pi}{l_P^2} + 2\ln r\right) - \frac{4\pi r^2}{l_P^2}\right\}\right]. \tag{9}$$

The first derivative of $\ln T_{\text{mod}}$ over $\ln r$ gives

$$\frac{d\ln T_{\text{mod}}}{d\ln r} = 3 + \frac{d\ln\rho}{d\ln r} - \frac{2 - \frac{8\pi r^2}{l_P^2}}{\frac{\pi\beta_0}{4}\ln\left(\frac{\pi r^2}{l_P^2} - \frac{4\pi r^2}{l_p^2}\right)}. \tag{10}$$

Thus, mass of galaxy clusters within EiBI-GUP, $M_{EiBI-GUP}$, can be calculated, i.e.

$$\frac{M_{EiBI-GUP}}{\mathcal{M}_\odot} = \frac{k_B T_{mod} r}{G\mu m_p}\left(\frac{d\ln\rho}{d\ln r} + \frac{d\ln T_{mod}}{d\ln r}\right)\left[1 + \xi\left\{\frac{M^2 G\hbar}{\pi r^2 c} + \frac{\hbar^2}{16r^2}\ln\left(\frac{\pi r^2}{l_P^2}\right)\right\}\right]^{-1} - \frac{r^2\kappa}{4}\frac{d\rho}{dr}. \tag{11}$$

Here, μ is the average molecular weight of the galaxy cluster, m_p denotes the mass of the proton, M is the unmodified mass of the galaxy cluster, $\xi = \frac{\beta_0\hbar^2}{l_P^2}$, $M_\odot$ is the mass of the sun, and $\mathcal{M}_\odot = 10^{14}M_\odot$. Meanwhile, the mathematical expression of $\frac{d\ln\rho}{d\ln r}$ can be referred to EQUATION (C1) in Rahvar & Mashhoon [34].

Numerical Results

For the numerical calculations, we chose $\beta_0 = -1.656 \times 10^{10}$ and $\kappa = 5.8 \times 10^{40}$. The numerical calculations for the mass of 12 galaxy clusters are shown in TABLE 1. The data for

r_{500} and $M_{\text{Newtonian}}$ were obtained from Gupta & Desai [9]. In this study, the variation in β_0 does not have a significant impact, where all galaxy cluster masses are on the order of $10^{-10}M_{\odot}$, which is vastly different from $M_{\text{Newtonian}}$ in the order of $10^{14}M_{\odot}$. Additionally, the numerical simulations also show that such a significant reduction in mass is dominated by the factor β_0, so the parameter κ does not have a significant impact.

TABLE 1. Galaxy clusters data within EiBI-GUP theory.

Galaxy clusters' name	r_{500} **(kpc)**	$M_{EiBI-GUP}$ (10^{-19} $M_{\odot}$)	$\frac{\Delta M}{M_{\text{Newtonian}}}$ (%)
A133	966	2.81	100
A262	629	2.24	100
A383	912	3.72	100
A478	1279	4.46	100
A907	1066	4.63	100
A1413	1256	4.67	100
A1795	1199	3.79	100
A1991	699	2.42	100
A2029	1319	5.02	100
A2390	1368	6.93	100
MKW4	606	1.40	100
RXJ1159	643	1.30	100

From TABLE 1, we can see that the galaxy cluster masses obtained in the EiBI-GUP theory are not relevant to the actual physical conditions, considering that generally, galaxy cluster masses should be on the order of $10^{14}M_{\odot}$, similar to $M_{\text{Newtonian}}$. Thus, all corrections to the galaxy cluster masses in the EiBI-GUP theory compared to the Newtonian masses of the galaxy clusters are 100%. This is due to the order of magnitude difference between the two types of masses being too large, so the difference between the Newtonian mass and the EiBI-GUP mass ΔM tends to be the same as the Newtonian mass itself.

The difference between the galaxy cluster masses obtained from calculations and those from observations is indeed possible, but generally not to such a large order of magnitude. For example, please refer to Desai [35], where there is a difference in galaxy cluster masses between the calculations using the hydrostatic equation and the observations of the gravitational lensing phenomenon.

Indeed, several theories have been developed to explain the differences in mass values. A very popular theory is the existence of dark matter in galaxy clusters, although the existence of this matter can be disputed by other theories[34]. Theories that dispute the existence of dark matter include the non-local gravity theory [34], beyond Horndeski gravity [36], and the EiBI theory [20]. The EiBI-GUP theory is actually a potential theory to dispute the existence of dark matter, considering that the results from the pure EiBI theory for galaxy clusters previously derived by Apryandi and Pattersons have already provided quite good results [19]. However, through the numerical simulations presented in TABLE 1, it is shown that the EiBI-GUP theory is not sufficient to solve the problem of the mass discrepancy in galaxy clusters, nor is it sufficient to dispute the existence of dark matter.

In this study, a linear fitting was also performed between the galaxy cluster masses in the EiBI-GUP theory and the baryonic mass of the galaxy clusters, which is the galaxy cluster mass, without considering the presence of dark matter. The baryonic mass data was obtained from Rahvar and Mashhoon [34]. The result of this linear fitting is shown in FIGURE 1. There is a difference between the galaxy cluster masses calculated using the Newtonian hydrostatic equation and the baryonic mass of the galaxy clusters obtained through observation. By assuming the absence of dark energy and dark matter, the comparison between the baryonic mass of the galaxy clusters and the galaxy cluster masses in the EiBI-GUP theory is shown on the horizontal axis in FIGURE 1.

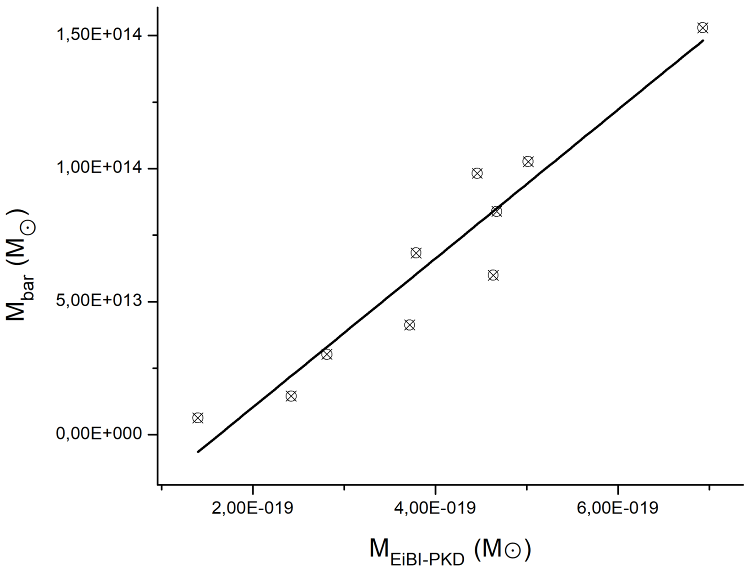

FIGURE 1. Linear fitting comparison of galaxy cluster masses in the EiBI-GUP theory with baryonic mass.

The slope of the linear fitting line is $(2.796 \pm 0.307) \times 10^{32}$. As with the results we obtained from TABLE 1, the slope value in FIGURE 1 is also not physically relevant and does not address the mass discrepancy problem between the mathematical calculations and the baryonic mass. In general, the EiBI-GUP theory is not physically relevant for application to galaxy cluster objects. Then, the unphysical mass reduction of ~33 orders of magnitude indicates overshoot, revealing that the model's GUP with EiBI corrections—inherently quantum-gravitational and pertinent to the Planck scale ($\sim 10^{-35}$ m)—are being inappropriately extrapolated to the classical astrophysical regime of galaxy clusters ($\sim 10^{23}$ m), thus necessitating a rigorous investigation to identify the constrained parameter space where GUP effects are a subtle, perturbative correction rather than a dominant force.

To validate the results obtained, an exponential fitting was also performed between the galaxy cluster masses in the EiBI-GUP theory and the baryonic masses of the clusters. This fitting yielded the following mathematical function:

$$M_{bar} = p_1 e^{-\frac{M_{EiBI-GUP}}{p_2}} + p_3 + p_4 M_{EiBI-GUP}\,, \tag{12}$$

where $p_1 = 5.22 \times 10^{15} \pm 1.61 \times 10^{18}$, $p_2 = -4.74 \times 10^{-18} \pm 6.98 \times 10^{-16}$, $p_3 = -5.25 \times 10^{15} \pm 1.61 \times 10^{-18}$, and $p_4 = -9.21 \times 10^{32} \pm 1.71 \times 10^{-35}$. It can be observed that the fittings shown in FIGURE 1 and FIGURE 2 exhibit very large uncertainties, rendering these results highly irrelevant to the actual physical conditions. Based on all the numerical results obtained, it can be concluded that the mass formulation within the EiBI-GUP theory is not suitable for application to galaxy cluster objects. It is worth noting that the present limitation of the EiBI–GUP framework for galaxy clusters does not rule out its potential relevance in other astrophysical settings. In particular, compact stars provide an intriguing arena where modified gravity may play an important role. For example, the secondary compact object in the GW190814 event, with an estimated mass of $2.50 - 2.67\, M_{\odot}$ [37], challenges the standard understanding of the maximum neutron star mass within general relativity and conventional equations of state. This tension could be alleviated in modified gravity scenarios such as EiBI, suggesting that further investigations of the EiBI–GUP framework in the context of neutron stars and related compact objects may provide valuable insights.

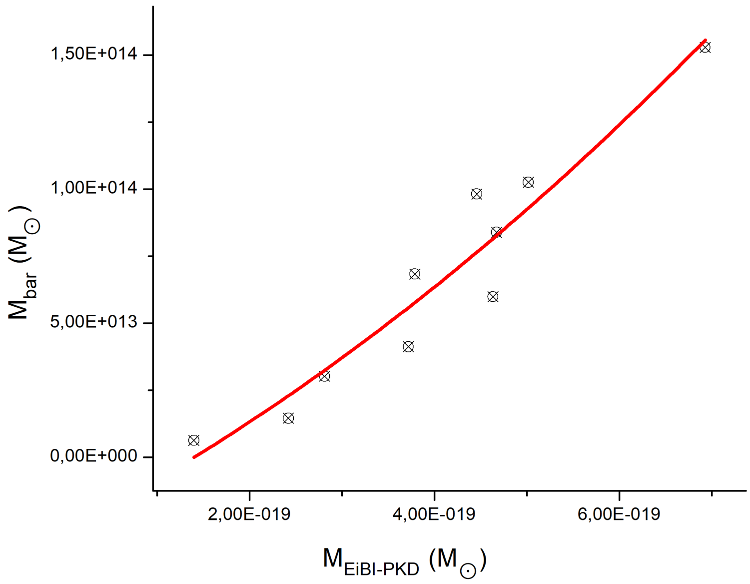

FIGURE 2. Exponential fitting of the comparison between galaxy cluster masses in the EiBI-GUP theory and baryonic masses.

CONCLUSION

Our main contribution is to investigate the impact of the GUP within the framework of EiBI gravity on the mass of galaxy clusters. This study provides a novel perspective on how quantum-gravity–motivated corrections may affect astrophysical systems, and it points toward further refinements and comparisons with other approaches in future work. The mass formulation of galaxy clusters in the EiBI-GUP theory has been successfully derived. This formulation was applied to 12 galaxy clusters. The mass corrections in the EiBI-GUP theory were found to be physically irrelevant when compared to the Newtonian mass. Additionally, the mass corrections in the EiBI-GUP theory do not resolve the discrepancies in baryonic mass. Based on the numerical calculations, it can be concluded that the EiBI-GUP theory is not suitable for application to galaxy cluster objects. Also based on these findings, the failure of EiBI-GUP to produce realistic masses for galaxy clusters suggests a critical limitation, modified gravity theories must produce negligible deviations from General Relativity on galactic and cluster scales to remain consistent with observational data. This implies that such theories are either highly constrained in their parameter space or that their significant effects are confined to regimes of even stronger gravity, such as the very early universe or the immediate vicinity of black hole singularities.

Although the mass formulation in the EiBI-GUP theory cannot be applied to galaxy clusters, it is possible that this formulation could be applicable to other more compact astronomical objects, such as white dwarfs, neutron stars, and quark stars. Additionally, galaxy cluster mass calculations could be explored using alternative untested theories, such as a combination of Rastall gravity and GUP or beyond Horndeski gravity (BHG) coupled with GUP. Although the EiBI–GUP framework does not yield significant improvements in the case of galaxy clusters, this approach may still be promising for applications to compact stars, where modified gravity effects could play a more relevant role.

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