BOUND STATE SOLUTION OF DIRAC EQUATION FOR SCARF POTENTIAL WITH NEW TENSOR COUPLING POTENTIAL FOR SPIN AND PSEUDOSPIN SYMMETRIES USING NIKIFOROV-UVAROV METHOD

U.A. Deta[1*)], A. Suparmi[2], and C. Cari[2]

[1]Jurusan Fisika, FMIPA, Universitas Negeri Surabaya

Jl. Ketintang, Surabaya 60231

[2]Jurusan Fisika, FMIPA, Universitas Sebelas Maret

Jl. Ir. Sutami No. 36A Kentingan, Surakarta 57126

*) Email: utamaalan@fmipa.unesa.ac.id

Abstract

Bound state solution of Dirac Equation for trigonometric Scarf potential with a new tensor coupling under spin and pseudospin symmetric limits are investigated using Nikiforov-Uvarov method. The new tensor potential proposed is inspired by superpotential form in SUSY quantum mechanics. The Dirac equation with trigonometric Scarf potential coupled by new tensor potential for the pseudospin and spin symmetric cases reduces to Schrodinger type equation with shape invariant potential since the proposed new potentials are similar to the superpotential of Scarf potential. The relativistic wave functions are exactly obtained eigenfunction of NU method in terms of Jacobi polynomials and the relativistic energy equation is exactly obtained by using eigenvalue of NU method in the approximation scheme of centrifugal term. The new tensor potential omits the energy degeneracy both for pseudospin and spin symmetric cases.

Keywords: Scarf potential, new tensor coupling potential, spin and pseudospin symmetry, Nikiforov-Uvarof methods.

1. Introduction

The exact analytical solutions of Dirac equations play important roles in relativistic quantum mechanics since they provide all important information of the system investigated. To describe the motion of spin half particles, some authors have explored the Dirac equations whose have exact solution under approximation scheme of centrifugal term for various potentials with tensor potentials [1-10]. From the observation, the expression of the tensor coupling potentials under the approximation scheme of centrifugal term are similar to the expression of the component of the given potentials.

Dirac equation for central/non-central potentials have been solved mostly by Nikiforov-Uvarof (NU) method [4, 9-13], factorization methods and SUSY QM [14-16], hypergeometric and confluent hypergeometric method [17-22], and asymptotic iteration method [23-24], Romanovski Polinomials [25-27], in the limit of spin and pseudospin symmetries. However, there are only few potentials that are solved exactly such as coulomb and harmonics oscillator potentials with Coulomb–type tensor potential, but other potentials are solvable only for *s*-wave. For *l*-wave, the Dirac Equations for central potentials are only solved approximately due to the contribution of the centrifugal term. The approximation scheme of the centrifugal term was proposed by Greene and Aldrich [28] and this approximation works well for trigonometric, hyperbolic and exponential potentials.

The new tensor potential is proposed due to the inspiration of the algebraic structure of SUSY quantum mechanics whose super partner potential is composed of square of the superpotential and its derivative [27]. The proposed new tensor potential is trigonometric cotangent plus cosecant potential which is similar to the superpotential form of trigonometric Scarf potential. We have solved this potential using Romanovski Polinomials [27]. In this paper, we will solve this new tensor potential using another method, that is, Nikiforov-Uvarov method.

The new tensor coupling potential as a function of trigonometric expression given as [27]

$$U(r) = -a(V_2 \cot ar + V_3 \csc ar) \quad (1)$$

with V_2 and V_3 are the strength of the nucleon forces, and a is the parameter that control the range of the tensor potential. The negative trigonometric cotangent potential alone is similar with the combination of Coulomb potential with square well potential therefore it is expected that combination of trigonometric cotangent and cosecant potential is suitable to be a screening potential as Coulomb-type and Yukawa-type tensor potentials. These tensor potentials were originally used to model strong nuleon-nucleon interactions caused by the exchange in nuclear physics [28-30].

The motion of nucleon with mass M in a repulsive vector potential and an attractive scalar potential plus a tensor potential $U(r)$ is described by Dirac equation given as [9,12,19, 23]

$$\{\vec{\alpha}.\vec{p} + \beta(M + V_S(r)) - i\beta\vec{\alpha}.\hat{r}U(r)\}\psi(\vec{r}) = \{E - V_V(r)\}\psi(\vec{r}) \quad (2)$$

where E is the relativistic energy and $\vec{p}$ is the three dimensional momentum operator, $-i\nabla$,

$$\vec{\alpha} = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix},\ \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad (3)$$

with σ is three dimensional Pauli matrices, I is 2×2 identity matrix. By taking $\hbar = 1$, c =1 and writing the Dirac spinor in Eq.(2) as

$$\psi(\vec{r}) = \begin{pmatrix} \zeta(\vec{r}) \\ \phi(\vec{r}) \end{pmatrix} = \begin{pmatrix} \dfrac{F_{n\kappa}(r)}{r} Y_{jm}^{l}(\theta,\varphi) \\ i\dfrac{G_{n\kappa}(r)}{r} Y_{jm}^{\tilde{l}}(\theta,\varphi) \end{pmatrix} \quad (4)$$

where $G_{n\kappa}(r)$ and $F_{n\kappa}(r)$ are the lower and upper components of Dirac spinors, respectively, $Y_{jm}^{l}(\theta,\varphi)$ is spin spherical harmonics, $Y_{jm}^{\tilde{l}}(\theta,\varphi)$ is pseudospin spherical harmonics, l is orbital quantum number, $\tilde{l}$ is pseudo orbital quantum number, and m is the projection of the angular momentum on the z-axis.

By inserting Eqs. (3) and (4) into Eq. (2), we get

$$\left(\frac{d}{dr} + \frac{\kappa}{r} - U(r)\right)F_{n\kappa}(r) = \left(M + E_{n\kappa} - V_V(r) + V_S(r)\right)G_{n\kappa}(r) \quad (5)$$

and

$$\left(\frac{d}{dr} - \frac{\kappa}{r} + U(r)\right)G_{n\kappa}(r) = \left(M - E_{n\kappa} + V_V(r) + V_S(r)\right)F_{n\kappa}(r) \quad (6)$$

By manipulating Eqs. (5) and (6) we obtain Dirac equations for pseudospin and spin symmetries, respectively, given as

$$\left\{\frac{d^2}{dr^2} - \frac{\kappa(\kappa-1)}{r^2} + \frac{2\kappa}{r}U(r) - U^2(r)\right\}G_{n\kappa}(r) + \left\{\frac{dU}{dr} + \Delta(r)\left(M - E_{n\kappa} + \Sigma(r)\right)\right\}G_{n\kappa}(r) = \left(M + E_{n\kappa}\right)\left(M - E_{n\kappa} + \Sigma(r)\right)G_{n\kappa}(r) \quad (7)$$

and

$$\left\{\frac{d^2}{dr^2} - \frac{\kappa(\kappa+1)}{r^2} + \frac{2\kappa}{r}U(r) - U^2(r)\right\}F_{n\kappa}(r) - \left\{\frac{dU}{dr} + \Sigma(r)\left(M + E_{n\kappa} - \Delta(r)\right)\right\}F_{n\kappa}(r) = \left(M + E_{n\kappa} - \Delta(r)\right)\left(M - E_{n\kappa}\right)F_{n\kappa}(r) \quad (8)$$

where $\Sigma(r) = V_V(r) + V_S(r)$ is the sum of scalar and vector potentials, and $\Delta(r) = V_V(r) - V_S(r)$ is the different between vector and scalar potentials.

In the case of pseudospin symmetry, from Eq. (7) we have $\kappa(\kappa-1) = \tilde{l}(\tilde{l}+1)$ that gives

$$\kappa = -\tilde{l} = -(j+\tfrac{1}{2}) \to \tilde{l} = l+1 \text{ and } j = l + \tfrac{1}{2} \quad (9)$$

$$\kappa = \tilde{l}+1 = (j+\tfrac{1}{2}) \to \tilde{l} = l-1 \text{ and } j = l - \tfrac{1}{2} \quad (10)$$

for $\kappa < 0$ and for $\kappa > 0$, respectively, κ is spin orbit quantum number. For pseudospin symmetry the sum and the different between vector and scalar potentials are given as

$$\Sigma(r) = C_{ps} \text{ and } \Delta(r) = V(r) \quad (11)$$

where C_{ps} is constant. Eq. (7) is Schrodinger-type equation with the effective potential V_{eff}

$$V_{ef} = \frac{\kappa(\kappa-1)}{r^2} - \frac{2\kappa}{r}U(r) + U^2(r) - \frac{dU}{dr} - V(r)\left(M - E_{n\kappa} + C_{ps}\right) = \varphi^2(r) - \varphi'(r) - V(r)\left(M - E_{n\kappa} + C_{ps}\right) \quad (12)$$

is shape invariant since the effective potential in Eq. (12) is combination of two potentials, $V_L = \phi^2(r) - \phi'(r)$ and $V_R = V(r)\left(M - E_{n\kappa} + C_{ps}\right)$ with $\varphi(r) = U(r) - \frac{\kappa}{r}$ is similar to the superpotential of the trigonometric Scarf potential in the approximation scheme of centrifugal term.

For spin symmetry we have $\kappa(\kappa+1) = l(l+1)$ that leads to the values

$$\kappa = -(l+1) = -(j+\tfrac{1}{2}) \to j = l + \tfrac{1}{2}, \text{ for } \kappa < 0 \quad (13)$$

and

$$\kappa = l = (j+\tfrac{1}{2}) \to j = l - \tfrac{1}{2}, \text{ for } \kappa > 0 \quad (14)$$

The different and the sum of vector and scalar potentials for spin symmetry are

$$\Delta(r) = C_s \text{ and } \Sigma(r) = V(r) \quad (15)$$

Similar to the argumentation of the pseudospin symmetry, the Schrodinger-type equation in Eq. (8) has shape invariant potential with the effective potential V_{eff} given as

$$V_{ef} = \frac{\kappa(\kappa+1)}{r^2} - \frac{2\kappa}{r}U(r) + U^2(r) + \frac{dU}{dr} + V(r)\left(E_{n\kappa} + M - C_s\right) = \varphi^2(r) + \varphi'(r) + V(r)\left(E_{n\kappa} + M - C_s\right) \quad (16)$$

Both Dirac equations for pseudospin and spin symmetries in Eqs. (7) and (8) are solved using Nikiforov-Uvarov Method. The NU method which was developed by Nikiforov-Uvarov [31]. This method based on solving the second order linear differential equations by reducing it to a hypergeometric type equation by a suitable change of variable. By using the eigenvalue of this method, we have the energy of the system. The wave functions exactly obtained using the eigenfunction of NU method in terms of Jacobi polynomials.

2. Methods of Analysis

The Dirac equation of any shape invariant potential can be reduced into hypergeometric type differential equation by suitable variable transformation [32-35]. The hypergeometric type differential equation, which is solved using NU method, is presented as:

$$\frac{\partial^2\Psi(s)}{\partial s^2}+\frac{\bar{\tau}(s)}{\sigma(s)}\frac{\partial\Psi(s)}{\partial s}+\frac{\bar{\sigma}(s)}{\sigma^2}\Psi(s)=0 \tag{17}$$

where $\sigma(s)$ and $\bar{\sigma}(s)$ are polynomials at most in the second order, and $\bar{\tau}(s)$ is first order polynomial. Eq. (17) can be solved using separation of variable method which is expressed as:

$$\Psi=\phi(s)y(s) \tag{18}$$

By inserting Eq. (18) into Eq. (17) we get hypergeometric type equation, that is:

$$\sigma\frac{\partial^2 y}{\partial s^2}+\tau\frac{\partial y}{\partial s}+\lambda y=0 \tag{19}$$

ϕ (s) is a logarithmic derivative whose solution obtained from condition:

$$\frac{\phi'}{\phi}=\frac{\pi}{\sigma} \tag{20}$$

while the function π (s) and the parameter λ are defined as:

$$\pi=\left(\frac{\sigma'-\bar{\tau}}{2}\right)\pm\sqrt{\left(\frac{\sigma'-\bar{\tau}}{2}\right)^2-\bar{\sigma}+k\sigma} \tag{21}$$

$$\lambda=k+\pi' \tag{22}$$

The value of k in Eq. (21) can be found from the condition that the expression under the square root of Eq. (21) must be square of polynomial which is mostly first degree polynomial and therefore the discriminate of the quadratic expression is zero. A new eigenvalue of Eq. (19) is:

$$\lambda=\lambda_n=-n\pi'-\frac{n(n-1)}{2}\sigma'' \; ; \; n=0,\,1,\,2,\,\dots \tag{23}$$

where

$$\tau=\bar{\tau}+2\pi \tag{24}$$

The new bound state energy is obtained using Eqs. (22) and (23). To generate the bound state energy and the corresponding eigenfunction, the condition that $\tau'<0$ is required. The solution of the second part of the wave function, y_n (s), which is connected to Rodrigues relation [36], is given as:

$$y_n(z)=\frac{C_n}{\rho(z)}\frac{d^n}{dz^z}\left\{\sigma^n(z)\rho(z)\right\} \tag{25}$$

where C_n is normalization constant, and the weight function $\rho(s)$ must satisfies the condition:

$$\frac{\partial(\sigma\rho)}{\partial s}=\tau(s)\rho(s) \tag{26}$$

The wave function of the system is therefore obtained from Eqs. (20), (25) and (26).

3. Result and Discussion

3.1. Solution of Pseudospin Symmetry

The trigonometric Scarf potential that will be coupled with new tensor potential is given as

$$V(r)=a^2\left\{\frac{V_0}{\sin^2 ar}-\frac{V_1\cos ar}{\sin^2 ar}\right\} \tag{27}$$

where $V_0$and V_1a positive parameter which describe the depth of the potential, a is a positive parameter which control the range of the potential, and $0<r<\infty$. By inserting Eqs. (1) and (27) into Eq. (7),and take approximaxion for centrifugal term, $\frac{1}{r^2}\cong\frac{a^2}{\left(\sin^2 ar\right)}$ [28], we obtain

$$\begin{aligned}&\left\{\frac{d^2}{dr^2}-\frac{a^2\left\{\kappa(\kappa-1)+2\kappa V_3+V_3^2+V_2^2-V_2-V_0\gamma_{ps}\right\}}{\sin^2 ar}\right\}G_{n\kappa}(r)\\&-\frac{a^2(2\kappa V_2+2V_2V_3-V_3+V_1\gamma_{ps})\cos ar}{\sin^2 ar}G_{n\kappa}(r)\\&=\left\{(M+E_{n\kappa})(M-E_{n\kappa}+C_{ps})-a^2V_2^2\right\}G_{n\kappa}(r)\end{aligned} \tag{28}$$

By setting

$$A_{ps}=\left\{\kappa(\kappa-1)+2\kappa V_3+V_3^2+V_2^2q-V_2q-V_0\gamma_{ps}\right\} \tag{29}$$

$$B_{ps}=(V_3-2\kappa V_2-2V_2V_3-V_1\gamma_{ps}) \tag{30}$$

$$E'_{ps}=\left\{\frac{(M+E_{n\kappa})(M-E_{n\kappa}+C_{ps})}{a^2}-V_2^2\right\} \tag{31}$$

$$\gamma_{ps}=\left(M-E_{n\kappa}+C_{ps}\right) \tag{32}$$

in Eq. (28) we obtain one dimensional Schrodinger-type equation given as 3

$$\left\{\frac{d^2}{dr^2}-\frac{a^2A_{ps}}{\sin^2 ar}+\frac{a^2B_{ps}\cos ar}{\sin^2 ar}\right\}G_{n\kappa}(r)=a^2E'_{ps}G_{n\kappa}(r) \tag{33}$$

By setting $\cos ar=s$ in Eq. (33) we get

$$\frac{d^2G_{n\kappa}(r)}{ds^2}-\frac{s}{(1-s^2)}\frac{dG_{n\kappa}(r)}{ds}-\left\{\frac{A_{ps}-B_{ps}s+E'_{ps}(1-s^2)}{(1-s^2)^2}\right\}G_{n\kappa}(r)=0 \tag{34}$$

By comparing Eqs. (17), (34), and using eigenvalue of NU method in Eq. 23, we obtain the relativistic energy of Dirac Equation is:

$$\begin{aligned}&\left\{\frac{(M+E_{n\kappa})(E_{n\kappa}-M-C_{ps})}{a^2}+V_2^2\right\}=\\&\left\{\tfrac{1}{2}\sqrt{\left(A_{ps}+\tfrac{1}{4}\right)+B_{ps}}-\tfrac{1}{2}\sqrt{\left(A_{ps}+\tfrac{1}{4}\right)-B_{ps}}+n+\tfrac{1}{2}\right\}^2\end{aligned} \tag{35}$$

The relativistic energy spectra calculated from relativistic energy equation in Eq. (35) are presented in Table 1. It shows that the degeneracy occurs for a pair of states (*n, l, j*+½) with (*n, l+2, j*-½). The degeneracy is removed by the presence of the trigonometric cotangent and cosecant tensor potential, as shown in the 5th and 9th columns. For $\kappa < 0$, the presence of the tensor potential decreases the relativistic energy while for $\kappa > 0$ the tensor potential increases the relativistic energy.

Table 1. Energy spectra for trigonometric Scarf potential with/without new tensor potential for V_0 = 4 fm^{-1}; V_1 = 3 fm^{-1}; a = 0.05 fm^{-1}; M = 3 fm^{-1}; and C_{ps} = -5 fm^{-1}.

l	n, $\kappa < 0$	(l, $j = l + ½$)	$E_{n\kappa} < 0$ $V_2 \& V_3 = 0$	$E_{n\kappa} < 0$ $V_2 = 0.6$, $V_3 = 0.8$	n, $\kappa > 0$	($l+2$, $j = l$ - ½)	$E_{n\kappa} < 0$ $V_2 \& V_3 = 0$	$E_{n\kappa} < 0$ $V_2 = 0.6$, $V_3 = 0.8$
0	0, -1	$0s_{1/2}$	-1.98997	-1.99674	0, 2	$0d_{3/2}$	-1.98997	-1.98058
1	0, -2	$0p_{3/2}$	-1.97773	-1.98834	0, 3	$0f_{5/2}$	-1.97773	-1.96487
2	0, -3	$0d_{5/2}$	-1.96107	-1.97518	0, 4	$0g_{7/2}$	-1.96107	-1.94500
0	1, -1	$1s_{1/2}$	-1.97756	-1.98768	1, 2	$1d_{3/2}$	-1.97756	-1.96427
1	1, -2	$1p_{3/2}$	-1.96091	-1.97485	1, 3	$1f_{5/2}$	-1.96091	-1.94457
2	1, -3	$1d_{5/2}$	-1.94022	-1.95741	1, 4	$1g_{7/2}$	-1.94022	-1.92107

By using eigenfunction of NU method in Eqs. (20), (25), and (26), we obtain the lower component of Dirac spinor for pseudospin symmetry, that is,

$$G_{n\kappa} = B_n\left(1-s^2\right)^{\left(\frac{1}{2}p+\frac{1}{4}\right)}\left(1+s\right)^{-\frac{B_{ps}}{4p}}\left(1-s\right)^{\frac{B_{ps}}{4p}} P_n^{(\alpha,\beta)}\left(s\right) \quad (36)$$

where $P_n^{(\alpha,\beta)}$ is Jacobi Polynomial, that is,

$$P_n^{(\alpha,\beta)}\left(s\right) = \frac{(-1)^n}{2^n n!}\left(1-s\right)^{-\alpha}\left(1+s\right)^{-\beta}, \frac{d^n}{dz^n}\left\{\left(1-s\right)^{\alpha}\left(1+s\right)^{\beta}\left(1-s^2\right)^n\right\} \quad (37)$$

$$\alpha = p - \frac{B_{ps}}{2p}, \quad \beta = p + \frac{B_{ps}}{2p}, \quad (38)$$

c, (39)

and B_n is normalization constant.

The ground state of lower component of Dirac spinor for any κ state from Eqs. (36) to (39) is

$$G_{0\kappa}(r) = \left(1-\cos ar\right)^{-\frac{1}{2}\sqrt{\left(A_{ps}+\frac{1}{4}\right)-B_{ps}}+\frac{1}{4}}\left(1+\cos ar\right)^{\frac{1}{2}\sqrt{\left(A_{ps}+\frac{1}{4}\right)+B_{ps}}+\frac{1}{4}} \quad (40)$$

The ground state wave function of upper component of Dirac spinor for pseudospin symmetry is obtained using Eqs. (6) and (40) in the approximation scheme of centrifugal term given as

$$F_{0\kappa}(r) = \left\{\begin{array}{l} \dfrac{a\left\{\frac{1}{2}\left(\sqrt{\left(A_{ps}+\frac{1}{4}\right)+B_{ps}} - \sqrt{\left(A_{ps}+\frac{1}{4}\right)-B_{ps}} + 1 - 2V_2\right)\cos ar\right\}}{\left(M - E_{n\kappa} + C_{ps}\right)} \\ -a\dfrac{\frac{1}{2}\sqrt{\left(A_{ps}+\frac{1}{4}\right)+B_{ps}} + \frac{1}{2}\sqrt{\left(A_{ps}+\frac{1}{4}\right)-B_{ps}} + (\kappa+V_3)}{\left(M - E_{n\kappa} + C_{ps}\right)} \end{array}\right\}$$

$$\left(1-\cos ar\right)^{-\frac{1}{2}\sqrt{\left(A_{ps}+\frac{1}{4}\right)-B_{ps}}-\frac{1}{4}}\left(1+\cos ar\right)^{\frac{1}{2}\sqrt{\left(A_{ps}+\frac{1}{4}\right)+B_{ps}}-\frac{1}{4}} \quad (41)$$

For exact pseudospin symmetry which occurs when $C_{ps} = 0$, the upper spinor in Eq. (41) exist if $M \neq E_{n\kappa}$ it means that there is no positive bound state energy for pseudospin symmetry.

3.2. Sollution of Spin Symmetry

The Dirac equation for spin symmetry is obtained by inserting Eqs. (1) and (27) into Eq. (8) given as

$$\begin{aligned} &\left\{\frac{d^2}{dr^2} - \frac{\kappa(\kappa+1)}{r^2} - \frac{2\kappa}{r}a\left(V_2\cot ar + V_3\csc ar\right) - a^2V_2\csc^2 ar\right\}F_{n\kappa}(r) \\ &-\left\{a^2\left(V_2\cot ar + V_3\csc ar\right)^2 + a^2V_3\csc ar\cot ar\right\}F_{n\kappa}(r) \\ &-a^2\left(\frac{V_0}{\sin^2 ar} - \frac{V_1\cos ar}{\sin^2 ar}\right)\left(M + E_{n\kappa} - C_s\right)F_{n\kappa}(r) \\ &= \left(M - E_{n\kappa}\right)\left(M + E_{n\kappa} - C_s\right)F_{n\kappa}(r) \end{aligned} \quad (42)$$

Using the approximation of the centrifugal term, $\frac{1}{r^2} \cong \frac{a^2}{\left(\sin^2 ar\right)}$ [28], into Eq. (42) we get

$$\begin{aligned} &\left\{\frac{d^2}{dr^2} - \frac{a^2\left\{\kappa(\kappa+1) + 2\kappa V_3 + V_3^2 + V_2^2 - V_2 + V_0\gamma_s\right\}}{\sin^2 ar}\right\}F_{n\kappa}(r) \\ &-\frac{a^2(2\kappa V_2 + 2V_2V_3 - V_3 - V_1\gamma_s)\cos ar}{\sin^2 ar}F_{n\kappa}(r) \\ &= \left\{\left(M - E_{n\kappa}\right)\left(M + E_{n\kappa} - C_s\right) - a^2V_2^2\right\}F_{n\kappa}(r) \end{aligned} \quad (43)$$

By setting

$$A_s = \left\{\kappa(\kappa+1) + 2\kappa V_3 + V_3^2 + V_2^2 - V_2 + V_0\gamma_s\right\} \quad (44)$$

$$B_s = (V_3 - 2\kappa V_2 - 2V_2V_3 + V_1\gamma_s) \quad (45)$$

$$E'_s = \left\{\frac{\left(M - E_{n\kappa}\right)\left(M + E_{n\kappa} - C_s\right)}{a^2} - V_2^2\right\} \quad (46)$$

$$\gamma_s = \left(M + E_{n\kappa} - C_s\right) \quad (47)$$

in Eq. (43) then Eq. (43) reduces to one dimensional Schrodinger-type equation given as

$$\left\{\frac{d^2}{dr^2} - \frac{a^2A_s}{\sin^2 ar} + \frac{a^2B_s\cos ar}{\sin^2 ar}\right\}F_{n\kappa}(r) = a^2E'_sF_{n\kappa}(r) \quad (48)$$

Eq. (48) are basically similar with Eq (33). Therefore, the relativistic energy and the wave functions for spin symmetry, both for upper and lower component of Diracspinors, is similar to the pseudospin symmetry limit. The only different is the values of *A*, *B*, and *C*. The ground state wave function of upper component of Dirac spinor for exact spin symmetry occurs when $C_s = 0$, the lower spinor exist if $M \neq -E_{n\kappa}$ therefore the system has no negative bound state relativistic energies.

4. Concluding Remark

The Dirac equation for trigonometric Scarf potential with trigonometric cotangent and cosecant tensor potential in the approximation scheme of centrifugal term is exactly solved using NUmethod both for pseudospin and spin symmetric cases. It was found to agree with previous works [27]. The trigonometric cotangent and cosecant tensor potential removes the degeneracy energies both for pseudospin and spin symmetries. The lower and upper component of Dirac spinors are obtained exactly in the approximation scheme of centrifugal term both for pseudospin and spin symmetries.

Acknowledgement

This research is partly supported by HibahPenelitiUtama (PUT) SebelasMaret University 2014 and DIKTI with contract number 165a/UN27.11/PN2013. The conference and publication fees supported by State University of Surabaya (Unesa).

References

[1] Alberto P, Lisboa R, Malheiro M, De Castro A S 2005 *Phys. Rev. C* **71** 034313-7
[2] Zhang LH, Li XP and Jia CS 2008 *Phys. Lett. A***372** 2201
[3] Wei GF and Dong SH 2008 *Phys. Lett. A***373** 49
[4] Ikhdair SM and Hamzavi M 2013 *Chin. Phys. B***22** 090305-1
[5] Ikhdair SM and Sever R 2010 *Appl. Math. Comput.* **216** 911
[6] Hamzavi M, Rajabi AA and Hassanabadi H 2010 *Phys. Lett. A***374** 4303
[7] Wei GF and Dong SH 2008 *Phys. Lett. A***373** 49
[8] Xu Y, He S and Jia CS 2008 *J. Phys. A: Math. andTheor.***41** 0255302-1
[9] Hamzavi M and Rajabi AA 2013 *Adv. High. En. Phys.***2013** 1
[10] Ikhdair SM and Sever R 2010 *arXiv: 1001. 4327v2* [math-ph]
[11] IkotAN 2013 *Commun. Theor Phys.***59** 268
[12] Aydogdu O, Maghsoodhi E and Hassanabadi H 2013 *Chin. Phys. B***22** 010302-1
[13] Zhou F, Wu Y and Guo JY 2009 *Commun. Theor. Phys.***52** 813
[14] Ping ZA and Chao W 2007 *High En. Phys. and Nucl. Phys.***31** 1027
[15] Chen G 2004 *Phys. Lett. A***328** 116-204
[16] Chen G, Chen ZD, Lou ZM 2004 *Chin. Phys.***13** 3279
[17] Guo JY, Zhou F, Gou FL and Zhou JH 2007 *Int. J. Mod. Phys. A***22** 4825
[18] Chen CH 2005 *Phys. Lett. A***339** 283
[19] Aydogdu O and Sever R 2011 *Phys. Lett. B.***703** 379
[20] Chen G 2004 *Mod. Phys. Lett. A***19** 26
[21] Yan Z and Gou JY 2008 *Chin. Phys. B***17** 0380
[22] Hu XQ, Luo G, Wu ZM, Niu LB and Ma Y 2010 *Commun. Theor. Phys.***53** 242
[23] Soylu A, Bayrak O and Boztosun I 2007 *J. Math. Phys.* **48** 082302
[24] Bahar MK and Yasuk F 2013 *Chin. Phys. B***22** 010301-1
[25] Cari C, Suparmi A, Deta UA, and S W Intan 2014 *MAKARA Journal of Science***17** 3
[26] Suparmi A and Cari C 2013 *At. Ind. J* **39** 1
[27] Suparmi A, Cari C, and Deta UA 2014 *Chin. Phys. B* (in press)
[28] Aydogdu O and Sever R 2010 *Few-Body Syst.***47** 193
[29] Akcay H 2009 *Phys. Lett.***373** 616
[30] Hamzavi M, Rajabi AA and Hassanabadi H 2010 *Few-Body Sys.***48** 171
[31] Nikiforov A F and Uvarov V B 1988 *Special Functions of Mathematical Physics*, (BirkhauserVerlag Basel) p. 317-318
[32] Suparmi 2011 *MekanikaKuantum II* (Surakarta: Physics Department-Faculty of MIPA-SebelasMaret University) pp. 166-169
[33] Ikot AN, Akpabio LE, and Uwah EJ 2011 *EJTP***8/25** 225-232
[34] Ikot AN, Antia AD, Akpabio LE, and Obu JA 2011 *JVR***6/2** 65-76
[35] Inomata A, Suparmi A, and Kurth S 1991 *Proceeding of 18th International Colloqium on Group Theoretical Methods in Physics V* 399
[36] Yasuk F, Berkdemir C, and Berkdemir R 2005 *J. Phys. A: Math. Gen***38** 65795.