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MIXING DISTRIBUTION ANALYSIS OF MIXTURE POISSON DISTRIBUTION FOR THIRD PARTY LIABILITY INSURANCE CLAIM FREQUENCY DATA IN INDONESIA

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ABSTRACT

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The Indonesian government plans to mandate third-party liability (TPL) insurance for all vehicle owners in 2025. However, statistical modeling of TPL claim frequency data in Indonesia has received limited attention in academic research. The mixture Poisson distribution can be considered as a distribution for TPL claim frequency data in Indonesia. This is because claim frequency data often experiences overdispersion. In this study, the mixing distribution of the mixture Poisson distribution for TPL claim frequency data in Indonesia will be analyzed using a bootstrap approach. The data used in this study is policyholder claim frequency data for comprehensive coverage of TPL for underwriting years 2015-2019 for vehicle categories 1, 2, 3, and 6 of PT. X in Indonesia. The results show that most distributions with more parameters have a larger p-value (more suitable for use as a mixing distribution for mixture Poisson distribution) than distributions with fewer parameters.

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1. INTRODUCTION

The level of vehicle ownership continues to increase in Indonesia. Based on the latest data from the Indonesian Motorcycle Industry Association (AISI), domestic vehicle sales in 2024 reached 6,333,310 units, an increase of 1.54% compared to 2023, which recorded 6,236,992 units. This increase shows a positive trend in the Indonesian vehicle industry [1]. Along with this increase, the number of traffic accidents also increased, as in 2023, Indonesia recorded 148,307 cases of traffic accidents, an increase of 0.06% compared to 2022, which recorded 140,248 cases. This increase shows an upward trend in the number of traffic accidents in Indonesia [2].

In the face of these risks, coverage is needed for all risks that occur for the owner of a vehicle. The presence of insurance can provide various forms of coverage according to the product. Vehicle insurance is a product that provides financial protection for vehicle owners against various risks, such as damage due to accidents, theft, or natural disasters. One type of insurance for vehicles is third-party liability (TPL). TPL insurance is an insurance product that provides compensation to third parties who are directly affected by the risks caused by the insured vehicle because of the risks guaranteed in the policy. TPL benefits can compensate for two things, namely death or injury to third parties involved in an accident and the cost of replacing damage to third-party assets outside of our assets as insurance policyholders. This means that the cost of repairing the third party will be borne by the insurance company.

The application of TPL insurance in vehicle insurance is increasingly becoming a concern along with the development of insurance policies in Indonesia. This policy aims to increase protection for vehicle owners and third parties who may suffer losses due to traffic accidents. During the Insurance Forum 2024 event held virtually on Tuesday, July 16, 2024, the Chief Executive of the Insurance, Guarantee, and Pension Fund Supervisor of the Financial Services Authority (OJK), Ogi Prastomiyono, announced that starting January 2025, the Indonesian government will require all vehicle owners, both cars and motorcycles, to have TPL insurance [3]. In implementing this policy, OJK is working with the Ministry of Finance and the Fiscal Policy Agency in drafting a government regulation (PP) governing TPL insurance. The obligation to have TPL insurance for vehicles is based on Law No. 4 of 2023 concerning Strengthening and Development of the Financial Sector (PPSK Law), which was inaugurated on January 12, 2023. In Article 39A, paragraph (1), of Chapter VI of the PPSK Law on Insurance, it is stated that the government has the authority to establish a compulsory insurance program in accordance with the needs. In addition, based on OJK Regulation (POJK) No. 69/2016, the implementation of compulsory insurance programs must be carried out competitively and can be run individually or in the form of a consortium.

In the implementation of TPL insurance, the calculation of pure premium contribution is an equally important aspect. The pure premium is the amount of money that must be paid by the insurance participant to cover the loss covered by the insurance, excluding other insurance costs, and is purely based on the risk of loss of the insurance participant. An accurate pure premium calculation is important to ensure that the amount of premium charged to policyholders reflects the actual level of risk. This allows insurance companies to maintain a financial balance between premium income and claim payments, thus avoiding the risk of losses due to higher-than-expected claims or overpricing that can reduce the attractiveness of insurance products for the public [4]. In addition, the calculation of the right pure premium also plays a role in maintaining the sustainability of the insurance industry [5].

In determining the pure premium contribution of TPL insurance products, data such as claim frequency data and claim size data are required. The data is then analyzed to identify the appropriate distribution [6]. The calculation of pure premium can be obtained through a parametric approach, so the selection of a suitable distribution for claim frequency data and claim size data is very important. Common distributions used to model claim frequency data are Poisson, binomial, negative binomial, and geometric distributions. Although these distributions are widely used in claims frequency modeling, in some cases, claims frequency data exhibit characteristics that cannot be fully explained by a single distribution. For example, when overdispersion occurs, which is a situation where the variance of the data exceeds its mean. The Poisson distribution alone may not be flexible enough to capture such data variations. So, an approach is needed by mixing with the distribution of its parameters, or what is called a mixture distribution. One of the distributions for claim frequency data included in the mixture distribution is the mixture Poisson distribution. The mixture Poisson distribution is often used as an alternative distribution for modeling claim frequency data when overdispersion occurs [7]. Some mixture Poisson distributions are Poisson-Lindley [8], Poisson-Inverse Gaussian [9], Generalized Poisson-Lindley [10] [11], Poisson-Sujatha [12] [13], Poisson-Amarendra [14] [15], and Poisson-Aradhana [7] [16].

In this study, the mixing distribution of the mixture Poisson distribution will be analyzed by performing bootstrap resampling on the claim frequency data. In each bootstrap sample, the estimated Poisson distribution parameters are calculated. Mixing distribution can be formed from the estimated values of Poisson distribution parameters of each bootstrap sample. The data that will be used in this study is policyholder claim frequency data for comprehensive coverage with TPL for the underwriting years 2015–2019 for vehicle categories 1, 2, 3, and 6 of PT. X in Indonesia. This study provides recommendations for several mixing distributions of the mixture Poisson distribution for TPL claim frequency data in Indonesia.

2. METHODS

Material and Data

The data used in this study are policyholder claim frequency data for comprehensive coverage with TPL for the underwriting years 2015–2019 and for vehicle categories 1, 2, 3, and 6 [17] from PT. X in Indonesia.

Research Method

Distributions for Claim Frequency Data

Claim frequency indicates the number of claims filed by policyholders in a given period. The selection of an appropriate distribution to model claim frequency is important because it describes the pattern of future claim occurrence [18]. In general, the distribution used to model claim frequency should reflect the main patterns in the claims data. Claim frequency data in insurance is often modeled with the Poisson distribution.

Poisson Distribution

The Poisson distribution is a discrete distribution with one parameter that defines the mean and variance of the distribution with the parameter λ. The Poisson distribution is used to determine the probability of an event in a particular time and place that is expected to occur very rarely. The probability mass function of the Poisson distribution is defined as follows [18]:

$$f_{X|\Lambda}(x|\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}; x = 0,1,2,\dots;\ \lambda > 0 \tag{1}$$

The Poisson distribution is a distribution that has equidispersion properties where the average value is equal to the variance value:

$$E(X|\lambda) = \lambda \tag{2}$$

$$V(X|\lambda) = \lambda \tag{3}$$

Poisson distribution is widely used in various fields. including insurance. However, in practice there is often a violation of the Poisson distribution properties, where the variance of the response variable is smaller than the average (underdispersion) or the variance is greater than the average (overdispersion) [19]. In some studies, many cases of overdispersion were found. In overcoming overdispersion cases, some modeling is formed that combines the Poisson distribution with several distributions, both discrete and continuous (mixture Poisson distribution) [20].

Mixture Poisson Distribution

Theorem 1. Suppose a random variable X has a conditional probability density function $f_{X|\Lambda}(x|\lambda)$ and a cumulative distribution function (cdf) $F_{X|\Lambda}(x|\lambda)$, where λ is a parameter of X. Suppose also that Λ is a random variable with density function $f_{\Lambda}(\lambda)$. Then the marginal density function of X is [18]:

$$f_X(x) = \int f_{X|\Lambda}(x|\lambda) f_{\Lambda}(\lambda) d\lambda \tag{4}$$

The marginal distribution function of X is:

$$F_X(x) = \int_{-\infty}^{x} \int f_{X|\Lambda}(y|\lambda) f_{\Lambda}(\lambda)\, d\lambda dy = \int F_{X|\Lambda}(x|\lambda) f_{\Lambda}(\lambda) d\lambda \tag{5}$$

Equations (4) and (5) represent the probability density and cumulative distribution functions of an infinite mixture distribution.

The expected value of the $k-th$ moment of X can be calculated as:

$$E\left(X^k\right) = E\left[E\left(X^k|\Lambda\right)\right] \tag{6}$$

Meanwhile, the variance of X can be calculated as:

$$Var(X) = E[Var(X|\Lambda)] + Var[E(X|\Lambda)] \tag{7}$$

The proof of **Theorem 1**, along with the derivation of equations (4)–(7), follows the formulation of infinite mixture distributions and can be found in Klugman et al. (2012) [18]. In **Theorem 1** above, the distribution of the random variable Λ, denoted by $f_{\Lambda}(\lambda)$, is known as the mixing distribution. If the conditional probability density function, $f_{X|\Lambda}(x|\lambda)$, in **Theorem 1** is a Poisson distribution, then the distribution of X, denoted by $f_X(x)$ is called the mixture Poisson distribution. Table 1 presents mixture Poisson distributions for several mixing distributions [21].

Table 1. Mixture Poisson Distribution for Several Mixing Distributions

Mixing Distribution	Mixture Poisson
Dirac	Poisson
Gamma, Erlang	Negative Binomial
Exponential	Geometric
Inverse Gaussian	Sichel
Poisson	Neyman
Generalized inverse Gaussian	Poisson-generalized inverse Gaussian
Generalized gamma	Poisson-generalized gamma
Generalized Pareto	Poisson-generalized Pareto
Inverse-gamma	Poisson-inverse gamma
Log-normal	Poisson-log-normal
Lomax	Poisson-Lomax
Pareto	Poisson-Pareto
Pearson Family	Poisson-Pearson family
Truncated normal	Poisson-truncated normal
Uniform	Poisson-uniform
Shifted gamma	Delaporte
Beta with specified parameters	Yule

The bootstrap approach will be used to analyze the mixing distribution of the mixture Poisson distribution, with the following steps:

1. Perform bootstrap resampling of claim frequency data. The expected result in this step is to obtain a bootstrap sample of claim frequency data.
2. Calculate the estimated value of the Poisson distribution parameters for the bootstrap sample of claim frequency data in step 1. The expected result in this step is to obtain the estimated value of the Poisson distribution parameters for the bootstrap sample of claim frequency data.

3. Repeat steps 1and 2 1,000 times. The expected result in this step is to obtain 1,000 estimated values of the Poisson distribution parameters for each bootstrap sample. Mixing distribution can be formed from the estimated values of Poisson distribution parameters for each bootstrap sample.
4. Perform distribution fitting for the data of the estimated values of the Poisson distribution parameters for each bootstrap sample. The expected result in this step is to obtain several candidate distributions that may be suitable for the mixing distribution of the mixture Poisson distribution.
5. Provide recommendations for several distributions for the mixing distribution of the mixture Poisson distribution.

Figure 1 presents a flowchart of the mixing distribution analysis stages of the mixture Poisson distribution.

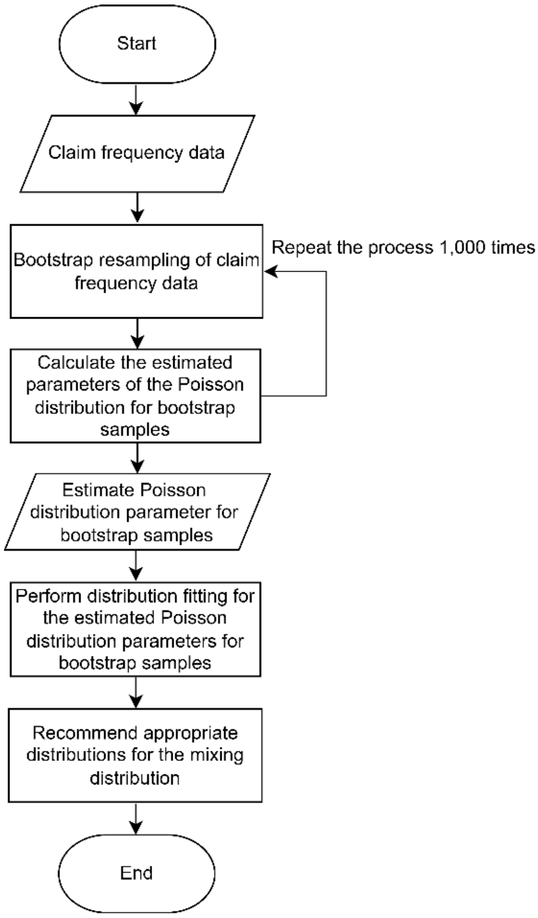

Figure 1. Flowchart of the mixing distribution analysis of the mixture Poisson distribution

3. RESULTS

Data Exploration Results

In this section, Table 2 presents the mean and variance of the claim frequency data, along with the percentage of policyholders who did not file a claim.

Table 2. Descriptive statistics

Category/Year		Mean	Variance	Percentage Not Claimed (%)
	2015	0.0025	0.0026	99.8
	2016	0.0025	0.0027	99.8
Category 1	2017	0.0027	0.0031	99.8
	2018	0.0029	0.0031	99.7
	2019	0.0029	0.0033	99.7
	2015	0.0029	0.0030	99.7
	2016	0.0032	0.0033	99.7
Category 2	2017	0.0034	0.0038	99.7
	2018	0.0034	0.0048	99.6
	2019	0.0042	0.0053	99.6
	2015	0.0038	0.0040	99.6
	2016	0.0037	0.0038	99.6
Category 3	2017	0.0034	0.0038	99.7
	2018	0.0037	0.0043	99.6
	2019	0.0044	0.0057	99.6
	2015	0.0060	0.0063	99.4
	2016	0.0059	0.0063	99.4
Category 6	2017	0.0049	0.0056	99.5
	2018	0.0049	0.0086	99.3
	2019	0.0070	0.0084	99.4

Based on the values in Table 2, it can be observed that

1. The sample variance of the claim frequency data is greater than the sample mean. This suggests that the claim frequency data for comprehensive coverage with TPL may exhibit overdispersion. Therefore, distributions capable of addressing overdispersion issues in the data are likely to be more appropriate. Mixture Poisson distribution may be suitable for this type of frequency data.
.
2. The proportion of policyholders who did not file a claim is very high (all above 99%). Therefore, distributions that are capable of handling a large number of zero values in the data may be more appropriate for the claim frequency data of comprehensive coverage with TPL, for example zero-inflated and zero-modified distributions.

Figure 2 presents histograms of the mixing distribution of mixture Poisson distribution for policyholder claim frequency data for comprehensive coverage with TPL for the underwriting years 2015–2019 for vehicle categories 1.

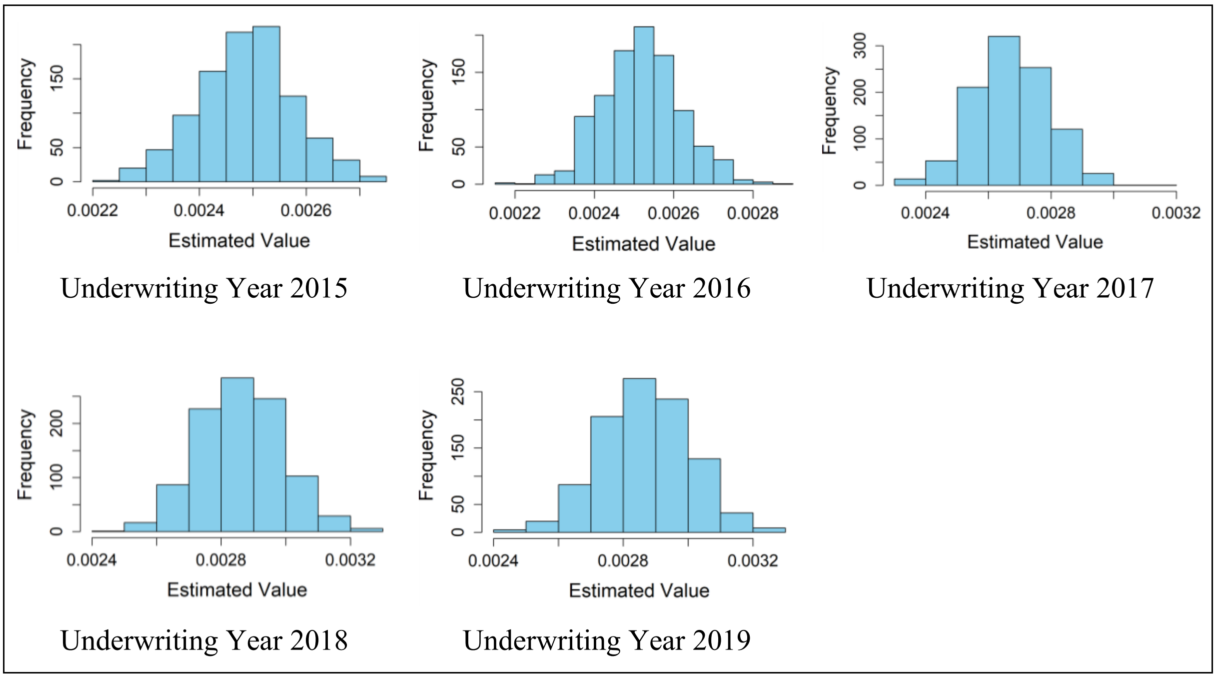

Figure 2. Mixing distribution of the mixture Poisson distribution for claim frequency data of comprehensive coverage of TPL expansion for underwriting years 2015-2019 vehicle category 1

Mixing Distribution of Mixture Poisson Distribution

Table 4 until Table 7 present the results of fitting 12 distributions to the data of bootstrap Poisson distribution parameter values for vehicle categories 1, 2, 3, and 6, respectively. Table 3 presents the probability density functions of the fitted distributions [22]. The distribution fitting is a way to analyze the mixing distribution of the mixture Poisson distribution.

Several of the distributions include extended versions with additional parameters, denoted as (3P) or (4P). Specifically, (3P) refers to a three-parameter version, and (4P) refers to a four-parameter version. These extensions improve model flexibility by incorporating additional shape or location parameters.

Table 3. List of Fitted Distributions

Distribution	Probability Density Function
Erlang	$f(x) = \frac{x^{m-1}}{\beta^m \Gamma(m)} exp\left(-\frac{x}{\beta}\right); 0 \leq x < +\infty$
Erlang (3P)	$f(x) = \frac{(x-\gamma)^{m-1}}{\beta^m \Gamma(m)} exp\left(-\frac{x-\gamma}{\beta}\right); \gamma \leq x < +\infty$
Gamma	$f(x) = \frac{x^{\alpha-1}}{\beta^\alpha \Gamma(\alpha)} exp\left(-\frac{x}{\beta}\right); 0 \leq x < +\infty$
Gamma (3P)	$f(x) = \frac{(x-\gamma)^{\alpha-1}}{\beta^\alpha \Gamma(\alpha)} exp\left(-\frac{x-\gamma}{\beta}\right); \gamma \leq x < +\infty$
Generalized Gamma	$f(x) = \frac{kx^{k\alpha-1}}{\beta^{k\alpha} \Gamma(\alpha)} exp\left(-\left(\frac{x}{\beta}\right)^k\right); 0 \leq x < +\infty$
Generalized Gamma (4P)	$f(x) = \frac{k(x-\gamma)^{k\alpha-1}}{\beta^{k\alpha} \Gamma(\alpha)} exp\left(-\left(\frac{x-\gamma}{\beta}\right)^k\right); \gamma \leq x < +\infty$
Inverse Gaussian	$f(x) = \sqrt{\frac{\lambda}{2\pi x^3}} exp\left(-\frac{\lambda(x-\mu)^2}{2\mu^2 x}\right); 0 < x < +\infty$

Inverse Gaussian (3P)	$f(x) = \sqrt{\frac{\lambda}{2\pi(x-\gamma)^3}} exp\left(-\frac{\lambda(x-\gamma-\mu)^2}{2\mu^2(x-\gamma)}\right); \gamma < x < +\infty$
Lognormal	$f(x) = \frac{exp\left(-\frac{1}{2}\left(\frac{lnx-\mu}{\sigma}\right)^2\right)}{x\sigma\sqrt{2\pi}}; 0 < x < +\infty$
Lognormal (3P)	$f(x) = \frac{exp\left(-\frac{1}{2}\left(\frac{ln(x-\gamma)-\mu}{\sigma}\right)^2\right)}{(x-\gamma)\sigma\sqrt{2\pi}}; \gamma < x < +\infty$
Weibull	$f(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} exp\left(-\left(\frac{x}{\beta}\right)^{\alpha}\right); 0 \leq x < +\infty$
Weibull (3P)	$f(x) = \frac{\alpha}{\beta}\left(\frac{x-\gamma}{\beta}\right)^{\alpha-1} exp\left(-\left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right); \gamma \leq x < +\infty$

The results of fitting 12 distributions to the bootstrap Poisson distribution parameter estimated values for vehicle category 1 (Table 4) show that.

1. All distributions are suitable as mixing distributions for mixture Poisson distribution, except the Weibull distribution.
2. All distributions with more parameters have a larger p-value (more suitable for use as a mixing distribution for mixture Poisson distribution) than distributions with fewer parameters in 2018.
3. Apart from 2018, all distributions with more parameters have a larger p-value (more suitable to be used as a mixing distribution for mixture Poisson distribution) than distributions with fewer parameters, except for the gamma and generalized gamma distributions and for lognormal in 2017.

Table 4. Mixing Distribution Fitting Results for Vehicle Category 1

Mixing Distribution	**P-Value**				
	2015	**2016**	**2017**	**2018**	**2019**
Erlang	0.1528	0.1052	0.0556	0.3732	0.3027
Erlang (3P)	0.2515	0.3704	0.5152	0.5142	0.9084
Gamma	0.3584	0.4725	0.4150	0.4758	0.8937
Gamma (3P)	0.2351	0.2473	0.1567	0.4868	0.8216
Generalized Gamma	0.3592	0.4721	0.4134	0.4706	0.8996
Generalized Gamma (4P)	0.1674	0.3280	0.2598	0.5610	0.8898
Inverse Gaussian	0.5619	0.3724	0.6539	0.2360	0.7000
Inverse Gaussian (3P)	0.5708	0.4750	0.7084	0.2379	0.8727
Lognormal	0.2754	0.3741	0.2940	0.6028	0.8390
Lognormal (3P)	0.4250	0.5125	0.2724	0.6133	0.8934
Weibull	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
Weibull (3P)	0.2722	0.1352	0.6569	0.5269	0.3665

The results of fitting 12 distributions to the bootstrap Poisson distribution parameter estimated values for vehicle category 2 (Table 5) show that.

1. All distributions are suitable as mixing distributions for mixture Poisson distribution, except the Weibull distribution.
2. All distributions with more parameters have larger p-values (more suitable for use as a mixing distribution for mixture Poisson distribution) than distributions with fewer parameters in 2017 and 2018.
3. Apart from 2017 and 2018, the lognormal, Weibull, gamma (2015), and generalized gamma (2015) distributions with more parameters have larger p-values (more suitable for use as a mixing distribution for mixture Poisson distribution) than distributions with fewer parameters.

Table 5. Mixing Distribution Fitting Results for Vehicle Category 2

Mixing Distribution	P-Value				
	2015	**2016**	**2017**	**2018**	**2019**
Erlang	0.7500	0.8764	0.6795	0.6738	0.3552
Erlang (3P)	0.7042	0.5757	0.9086	0.8216	0.1179
Gamma	0.8261	0.9757	0.7720	0.7830	0.4985
Gamma (3P)	0.8515	0.7119	0.9426	0.8787	0.3188
Generalized Gamma	0.8210	0.9753	0.7722	0.7837	0.4773
Generalized Gamma (4P)	0.9472	0.7817	0.8987	0.8950	0.2864
Inverse Gaussian	0.6627	0.9243	0.5795	0.5848	0.6994
Inverse Gaussian (3P)	0.6579	0.9050	0.5798	0.5855	0.4776
Lognormal	0.8705	0.9457	0.8578	0.8694	0.4329
Lognormal (3P)	0.9382	0.9693	0.9015	0.9098	0.5195
Weibull	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
Weibull (3P)	0.9791	0.6641	0.3986	0.3102	0.2700

The results of fitting 12 distributions to the bootstrap Poisson distribution parameter estimated values for vehicle category 3 (Table 6) show that.

1. All distributions are suitable as mixing distributions for mixture Poisson distribution, except the Weibull distribution.
2. All distributions with fewer parameters have larger p-values (more suitable as a mixing distribution for mixture Poisson distribution) than distributions with more parameters, except Weibull, Erlang (2016, 2017, 2019), inverse Gaussian (2016, 2017), and lognormal (2019).

Table 6. Mixing Distribution Fitting Results for Vehicle Category 3

Mixing Distribution	P-Value				
	2015	**2016**	**2017**	**2018**	**2019**
Erlang	0.6326	0.8961	0.5870	0.5577	0.5329
Erlang (3P)	0.2225	0.9209	0.7329	0.3162	0.8061
Gamma	0.7421	0.9027	0.8264	0.9255	0.6517
Gamma (3P)	0.5216	0.4687	0.5698	0.6687	0.3086
Generalized Gamma	0.7411	0.9029	0.8273	0.9282	0.6513
Generalized Gamma (4P)	0.5712	0.7465	0.7230	0.7691	0.4433
Inverse Gaussian	0.9001	0.9145	0.8564	0.8651	0.8517
Inverse Gaussian (3P)	0.8706	0.9152	0.9125	0.8591	0.8183
Lognormal	0.6488	0.8339	0.7305	0.8574	0.5462
Lognormal (3P)	0.5160	0.6728	0.6196	0.7519	0.5710
Weibull	< 0.0001	< 0.0001	< 0.0001	< 0.0001	0.0003
Weibull (3P)	0.5010	0.4678	0.6309	0.7208	0.8961

The results of fitting 12 distributions to the bootstrap Poisson distribution parameter estimated values for vehicle category 6 (Table 7) show that:

1. All distributions are suitable as mixing distributions for mixture Poisson distribution, except the Weibull distribution.
2. Weibull, lognormal, and Erlang distributions with more parameters have larger p-values (more suitable for use as a mixing distribution for mixture Poisson distribution) than distributions with fewer parameters, except for lognormal (years 2015, and 2017), and Erlang (years 2015 and 2019).
3. Gamma, generalized gamma, and inverse Gaussian distributions with fewer parameters have a larger p-value (more suitable for use as a mixing distribution for mixture Poisson distribution) than distributions with more parameters, except gamma (2019), generalized gamma (2019), and inverse Gaussian (2015, 2018).

Table 7. Mixing Distribution Fitting Results for Vehicle Category 6

Mixing Distribution	P-Value				
	2015	2016	2017	2018	2019
Erlang	0.4327	0.0165	0.0145	0.1570	0.8448
Erlang (3P)	0.2637	0.2389	0.0185	0.8252	0.7834
Gamma	0.5178	0.1686	0.0303	0.7033	0.4633
Gamma (3P)	0.2685	0.1187	0.0200	0.4666	0.9393
Generalized Gamma	0.5227	0.1748	0.0325	0.7017	0.4675
Generalized Gamma (4P)	0.4587	0.1522	0.0147	0.5047	0.7871
Inverse Gaussian	0.3091	0.3203	0.0758	0.9056	0.2753
Inverse Gaussian (3P)	0.4367	0.2610	0.0657	0.9349	0.2736
Lognormal	0.4348	0.1351	0.0237	0.5802	0.5780
Lognormal (3P)	0.4290	0.1753	0.0216	0.8741	0.7218
Weibull	0.0001	0.0010	0.0011	0.0003	< 0.0001
Weibull (3P)	0.6766	0.6365	0.6121	0.6816	0.0507

4. DISCUSSIONS

The results show that the claim frequency data for comprehensive coverage with TPL tends to experience overdispersion, which is characterized by a sample variance value that is consistently greater than the sample mean value in all years of coverage. This suggests that the use of the standard Poisson distribution is irrelevant, as it assumes that the mean and variance are equal. In addition, the high proportion of zero claims (more than 99%) indicates the presence of strong zero-inflation characteristics in the data, further emphasizing the need to use a more flexible distribution model.

The high p-values in the goodness-of-fit test results indicate that some of the distributions tested are suitable for use as mixing distributions in the mixture Poisson model. In particular, distributions with more parameters (such as lognormal and generalized gamma) tend to produce more stable and consistent p-values across different years and vehicle categories.

The results of this study are in line with previous studies that show that count data that experience overdispersion and zero-inflation are better modeled using a mixture distribution or a zero-inflated distribution. These studies support the use of flexible models in actuarial modeling.

5. CONCLUSION

Based on the data description and the results of the mixing distribution analysis, it can be concluded that the zero-inflated, zero-modified, and mixture Poisson distributions can be considered appropriate models for the claim frequency data of comprehensive coverage with TPL for vehicle categories 1, 2, 3, and 6 at PT. X in Indonesia. Among the twelve distributions evaluated in this study, eleven were found to be suitable as mixing distributions for the mixture Poisson distribution.

The analysis also shows that there are variations in distribution performance between vehicle categories. For vehicle categories 1 and 2, the distribution that has more parameters is more suitable to be used as a mixing distribution for the mixture Poisson distribution compared to the distribution with fewer parameters. Conversely, in vehicle category 3, the distribution with fewer parameters is more suitable as a mixing distribution for the mixture Poisson distribution. Meanwhile, in vehicle category 6, the number of matches for the distribution with fewer parameters as a mixing distribution for the mixture Poisson distribution is comparable to the distribution with a large number of parameters.

As an implication of the findings, the existing zero-inflated distribution, zero-modified distribution, and mixture Poisson distribution can be proposed as candidate models for the claim frequency data of comprehensive coverage with TPL for vehicle categories 1, 2, 3, and 6 at PT. X in Indonesia. In addition, new distributions can be proposed to model the data where the new distribution types are zero-inflated, zero-modified and mixture Poisson. The mixture Poisson distribution with its mixing distribution that does not yet exist in Table 1 can be proposed as a model for the claim frequency data of comprehensive coverage with TPL for vehicle categories 1, 2, 3, and 6 at PT. X in Indonesia. Mixing distributions that are not yet in Table 1 includes distributions with more parameters.

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