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SMALL-SAMPLE AND IMBALANCED DATA MODELING OF FAMILY QUALITY INDEX: A FIRTH LOGISTIC REGRESSION STUDY

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ABSTRACT

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Keywords:
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The Family Quality Index (FQI) in Indonesia is not published annually, limiting the availability of timely information for monitoring and evaluating family development policies. This study addresses this issue by identifying determinants of provincial FQI categories and developing a classification model that can be applied when official FQI measurements are unavailable. Secondary data from 34 provinces in Indonesia for 2023 were analyzed using Firth logistic regression, a method designed to reduce bias in small samples and imbalanced datasets. The response variable was the FQI category, classified into moderately responsive and responsive groups. Explanatory variables included mean years of schooling, open unemployment rate, poverty rate, and population density. The results show that poverty rate is the only predictor that remains statistically significant after bias correction, with higher poverty levels associated with a lower probability of a province being classified as responsive. The other variables were not statistically significant. Model performance was evaluated using Leave-One-Out Cross-Validation (LOOCV). Compared with conventional logistic regression estimated by maximum likelihood, the Firth model achieved higher accuracy (88.24% vs. 85.29%), Kappa (0.531 vs. 0.459), and sensitivity (0.931 vs. 0.897), while maintaining the same specificity. Additional sensitivity analyses using a reduced model produced similar results, indicating that the effect of poverty was stable across model specifications. These findings suggest that annually available socio-economic indicators may be used to provide provisional estimates of provincial FQI categories when official FQI data are unavailable, thereby supporting evidence-based family development policy evaluation.

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1. INTRODUCTION

Family quality, particularly in relation to gender empowerment and child protection, represents a critical issue in sustainable development in Indonesia, as it directly contributes to the achievement of the Sustainable Development Goals (SDGs), especially Goal 5 on gender equality [1]. The family serves as a fundamental unit of national development that influences human resource quality and the sustainability of social development. Therefore, the assessment of family quality must be conducted comprehensively to capture both structural and functional dimensions of family conditions. The Family Quality Index (FQI) is an official measurement tool used to assess family quality in Indonesia from the perspectives of gender equality and child protection [2]. The FQI consists of five dimensions, namely Structural Legality Quality, Physical Resilience, Economic Resilience, Social-Psychological Resilience, and Socio-Cultural Resilience. The government publishes FQI values at the provincial level and classifies provinces into specific categories as a basis for evaluating family development [2].

The government developed the FQI as a monitoring instrument for family development; however, it is not published on a regular annual basis due to limitations in data availability and cross-sectoral integration. This condition necessitates the use of statistical modeling approaches to generate timely and accurate predictions of FQI categories using relevant empirical predictors. Empirical FQI data at the provincial level form only two categories, namely responsive and moderately responsive, resulting in a binary response variable with a small sample size. Firth logistic regression provides a more reliable methodological solution than standard logistic regression under conditions of class imbalance and limited sample size [3]. This method modifies the likelihood function using a penalized likelihood based on Jeffreys prior to reduce bias in maximum likelihood estimation [4]. Introduced by Firth, this approach has been shown to produce stable parameter estimates and valid inference in binary response modeling [3], [5].

This study employs Firth logistic regression to model the Family Quality Index at the provincial level because the method produces unbiased parameter estimates in small-sample settings [3]. Firth logistic regression yields stable estimates in the presence of class imbalance in the response variable [3]. The method also produces well-defined coefficients when standard logistic regression suffers from separation or quasi-separation issues [4]. In addition, it provides more reliable variable inference through confidence intervals based on penalized likelihood. These advantages make Firth logistic regression particularly suitable for analyzing aggregated and limited public policy data such as provincial-level data [5].

Recent developments in Firth logistic regression demonstrate its increasing application in modeling categorical and ordinal data across various research domains. Studies in public health have applied this approach to model rare disease risk factors and have obtained more stable estimates compared to conventional logistic regression [6]. Meanwhile, issues related to family quality, gender equality, and child protection remain prevalent in Indonesia and show substantial variation across provinces. However, empirical studies focusing specifically on the Family Quality Index remain limited and tend to emphasize internal relationships among index dimensions without developing predictive models using external variables [7]. To date, no study has modeled provincial FQI categories in Indonesia using relevant external predictors for the prediction and identification of significant determinants. The lack of studies adopting this type of modeling approach limits the availability of empirical tools that can be used to estimate FQI categories when official FQI statistics are not yet available. It also restricts our understanding of the factors that distinguish family quality across provinces.

In response to this gap, the present study examines a set of provincial socio-economic indicators that are theoretically linked to family quality and applies Firth logistic regression to model FQI categories. Specifically, the study seeks to identify the factors associated with provincial FQI categories and to develop a classification model that can be used when official FQI measurements are unavailable. Firth logistic regression was selected because the data consist of a relatively small number of observations and an imbalanced distribution of categories. By relying on socio-economic indicators that are routinely published at the provincial level, the proposed model offers a practical approach for obtaining provisional estimates of FQI categories and may provide useful evidence for family development policy evaluation.

2. METHODS

This study adopts an explanatory quantitative approach aimed at examining the relationship between the response variable and predictor variables through inferential statistical modeling. A quantitative approach is selected because the study focuses on numerical analysis to identify factors influencing family quality in Indonesia. From a statistical perspective, this study is framed within the Generalized Linear Model (GLM), as the response variable follows a non-normal distribution and the relationship between the response and predictors is modeled using a link function [8]. The logit link function is employed because it is the canonical link for the binomial distribution and allows coefficient interpretation in terms of odds ratios, which are relevant for public policy analysis.

Data

The main object of this study is the Family Quality Index (FQI), developed and published by the Ministry of Women's Empowerment and Child Protection (MoWECP). The FQI is a composite measure of family quality constructed from five dimensions and twenty-nine indicators reflecting legal, physical, economic, social-psychological, and socio-cultural aspects [2]. The FQI ranges from 0 to 100 and is classified into three categories: less responsive, moderately responsive, and responsive to gender equality and child rights. However, exploratory analysis of provincial-level FQI data for 2023 indicates that the distribution forms only two categories, namely moderately responsive and responsive, with no observations in the less responsive category. Therefore, the FQI is treated as a binary variable, coded as 0 for moderately responsive provinces and 1 for responsive provinces, to ensure stable parameter estimation and appropriate model interpretation.

Overall, the study uses secondary data from 2023 with 34 provinces as the unit of analysis. Predictor variables are obtained from official publications of Statistics Indonesia (BPS) for the year 2023. The predictors include mean years of schooling, open unemployment rate, poverty rate, and population growth rate. These variables are selected because they are available annually at the provincial level and are not part of the FQI component indicators, thereby avoiding circularity bias in the modeling process. Table 1 presents the variables used in this study.

Table 1. Research Variables

Variable	Symbol	Scale / Unit	Data Source	Justification
Family Quality Index (FQI)	Y	Binary (moderately responsive, responsive)	MoWECP, FQI Report 2023	Official composite indicator of family quality and the main focus of the study
Mean Years of Schooling	X_1	Ratio (years)	BPS, Education Statistics 2023	Represents education level, which influences family quality and resilience [9][10]
Open Unemployment Rate	X_2	Ratio (%)	BPS, Sakernas 2023	Reflects labor market conditions and regional economic stability affecting family welfare [11][12]
Poverty Rate	X_3	Ratio (%)	BPS, Poverty Statistics 2023	Indicates economic pressure that directly affects family quality of life [13]
Population Density	X_4	(persons/km²)	BPS, Population Statistics 2023	Represents demographic dynamics influencing social and economic burdens on families [14]

Data Analysis Method

The main analysis employs binary logistic regression within the GLM framework, assuming that. $Y_i \sim Binomial\,(1, \pi_i)$ with a logit link function. The binary logistic regression model is expressed mathematically as follows:

$$\text{logit}\,(\pi_i) = \ln\left(\frac{\pi_i}{1-\pi_i}\right) = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + \beta_4 X_{4i}$$

where π_i denotes the probability that province i has a responsive family quality category. Model parameters are estimated using Maximum Likelihood Estimation (MLE). The regression coefficients are interpreted through odds ratios, $OR_k = \exp(\beta_k)$, which represent the change in odds associated with a one-unit increase in the predictor variable. However, a small sample size and class imbalance may introduce bias and separation issues in MLE estimation. Therefore, this study applies Firth logistic regression as a complementary approach [3]. This method modifies the log-likelihood function using a penalty based on Jeffreys prior. Mathematically, the penalized log-likelihood function is defined as:

$$\ell_F(\beta) = \ell(\beta) + \frac{1}{2}\log|\mathbf{I}(\beta)|$$

where $\ell(\beta)$ denotes the standard logistic regression log-likelihood function and $\boldsymbol{I}(\beta)$ represents the Fisher information matrix. This formulation produces estimators that are free from first-order bias and remain stable under small sample conditions [3], [4]. The analytical strategy involves estimating the MLE-based logistic model as a reference and the Firth model as a robustness check by comparing coefficient direction, odds ratio stability, and parameter significance. This approach was first introduced by Firth in 1993 and further developed by Heinze and Schemper in 2002 to address separation and instability issues in small samples.

Data Analysis Procedures

1. Family Quality Index (FQI) data and predictor variables are compiled from official publications of the Ministry of Women's Empowerment and Child Protection and Statistics Indonesia for 2023. This step includes verification of consistency in observation year, regional coverage, and indicator definitions across sources to ensure data validity [2].
2. Initial exploration is conducted to identify the distribution of the response categories and the characteristics of predictor variables. The proportion of FQI categories is examined to detect class imbalance that may affect the stability of MLE-based logistic regression estimates [4]. This step also includes checking value ranges and identifying extreme patterns in predictors.
3. The dataset is structured as a cross-sectional provincial dataset. Missing value checks are conducted to ensure completeness. No scale transformation is applied because all predictors are measured on a ratio scale and can be directly modeled using the logit function. The response variable is coded as binary, with 0 representing moderately responsive and 1 representing responsive provinces [2].
4. Descriptive analysis is conducted to summarize provincial characteristics using measures of central tendency and dispersion. This step provides contextual understanding of education, labor market conditions, poverty, and demographic dynamics across provinces prior to inferential modeling.
5. Pearson correlation is used to assess relationships among predictor variables to ensure the absence of strong multicollinearity and maintain the stability of regression estimates.
6. Binary logistic regression is estimated using maximum likelihood within the GLM framework [8]. This model serves as a baseline to evaluate coefficient direction, the magnitude of odds ratios, and potential estimation issues related to small sample size and class imbalance. Parameter significance is tested using the Wald test.
7. The MLE-based model is evaluated to detect indications of complete or quasi-separation. These issues are identified through extremely large coefficients, inflated standard errors, or model convergence failure [4]. This step justifies the use of penalized likelihood methods.
8. Parameter estimation in the Firth logistic regression
   The main estimation uses Firth logistic regression by maximizing the penalized likelihood $\ell_F(\beta) = \ell(\beta) + \frac{1}{2}\log|\mathbf{I}(\beta)|$ to reduce first-order bias and produce well-defined coefficients under small sample conditions [6]. Parameter estimation is performed using an iterative algorithm based on the penalized score function.
9. Estimated coefficients from the Firth model are interpreted using odds ratios and penalized likelihood-based confidence intervals. Interpretation focuses on the direction and substantive significance of predictors in distinguishing FQI categories across provinces.

10. Results from MLE and Firth logistic regression are compared to assess consistency in coefficient direction, stability of odds ratios, and changes in statistical significance. This step serves as a robustness check to ensure that conclusions do not depend on a specific estimation method.
11. Given the low events-per-variable (EPV) ratio and the limited number of provinces in the minority category, a sensitivity analysis is conducted using more parsimonious Firth logistic regression models. Alternative model specifications, including models with the theoretically strongest predictor(s), are estimated to evaluate whether the main substantive conclusions remain consistent across different model formulations.
12. Leave-One-Out Cross-Validation (LOOCV) is performed to assess the classification performance of the fitted models. Performance is evaluated using accuracy, sensitivity, specificity, and Cohen's Kappa. The LOOCV procedure provides a less optimistic assessment of model performance than apparent classification measures.
13. The Firth model is used to generate predicted probabilities of provincial FQI categories. Prediction focuses on the probability of a province being classified as responsive, supporting probabilistic policy interpretation rather than deterministic classification [5].
14. Results are interpreted by considering limitations related to small sample size, aggregated provincial data, and class imbalance. Interpretation emphasizes policy implications and the usefulness of the model as a rapid and informative predictive tool.

3. RESULTS

The data collection and verification stage resulted in a cross-sectional provincial dataset for 2023 consisting of 34 observations with one response variable and four predictor variables. All data on the Family Quality Index and provincial macro indicators were successfully verified from official sources of the Ministry of Women's Empowerment and Child Protection and Statistics Indonesia, with consistent definitions and regional coverage, ensuring their suitability for inferential analysis. Preliminary data exploration indicates that the distribution of Family Quality Index categories at the provincial level is imbalanced. A total of 29 provinces fall into the responsive category, while 5 provinces fall into the moderately responsive category. This condition confirms the presence of class imbalance in the response variable, which may affect the stability of maximum likelihood–based logistic regression estimates. Examination of the predictor variable ranges does not reveal any unreasonable extreme values.

The data preprocessing stage shows that all observations are complete with no missing values. All predictor variables are measured on a ratio scale and therefore do not require additional transformation for inclusion in the logit function. The response variable is then coded as binary according to the empirical data condition, with 0 representing moderately responsive provinces and 1 representing responsive provinces.

Table 2. Descriptive Analysis of Predictor Variables

Variable	Mean	Standard Deviation	Minimum	Median	Maximum	Range
Mean Years of Schooling (X_1)	8.93	0.91	7.15	8.90	11.45	4.30
Open Unemployment Rate (X_2)	4.61	1.42	2.27	4.32	7.52	5.25
Poverty Rate (X_3)	10.09	5.18	4.25	8.43	26.03	21.78
Population Density (X_4)	752.65	2749.12	10.00	102.50	16146.00	16136

The descriptive statistical analysis in Table 2 reveals substantial variation across provinces in all predictor variables, reflecting differences in education levels, labor market conditions, poverty pressure, and demographic dynamics. This variation provides substantive evidence that provincial macro-level factors may explain differences in family quality categories prior to inferential modeling. Pearson correlation analysis in Table 3 indicates the absence of strong correlations among predictor variables. This finding is further supported by the Variance Inflation Factor (VIF) analysis, where all VIF values were below 2 (ranging from 1.322 to 1.857), indicating that multicollinearity is not a concern in the fitted model. Therefore, all predictors can be retained in the regression model without causing instability in parameter estimation [8].

Table 3. Pearson Correlation among Predictor Variables

Variable	X_1	X_2	X_3	X_4
X_1	1.0000	0.4962	-0.4847	0.4906
X_2		1.0000	-0.3495	0.2842
X_3			1.0000	-0.2174
X_4				1.0000

Table 4. VIF Values for Predictor Variables

X_1	X_2	X_3	X_4
1.8566	1.3591	1.3363	1.3224

Binary logistic regression based on Maximum Likelihood Estimation (MLE) is used as the initial reference model. The estimation results in Table 5 indicate that none of the predictor variables are statistically significant at the five percent significance level. Relatively large standard errors appear in several coefficients, particularly for the intercept and the education variable. This condition suggests instability in parameter estimation due to a small sample size and imbalanced data.

Table 5. MLE-Based Binary Logistic Regression Estimates

	Estimate	Std. Error	z value	Pr(>|z|)
(*Intercept*)	8.4569	18.3170	0.462	0.644
X1	2.3714	4.2279	0.561	0.575
X2	-0.9555	2.3105	-0.414	0.679
X3	-1.6670	1.3559	-1.229	0.219
X4	-0.0010	0.0016	-0.659	0.510

Further evaluation of the MLE model indicates the presence of quasi-separation, as reflected in the large coefficients and standard errors for several predictor variables. This finding provides strong methodological justification for applying a penalized likelihood approach, as recommended in the logistic regression literature for sparse and imbalanced data [6].

The main estimation using Firth logistic regression, based on Table 6, shows a clear improvement in parameter stability and inference. The poverty rate variable exhibits a significant negative effect on the probability of a province being classified as responsive in the FQI, with a p-value of 0.0005 and a 95% confidence interval that does not include zero. Other variables show consistent directional effects but are not statistically significant.

Table 6. Penalized Likelihood–Based Binary Logistic Regression Estimates (Firth Model)

	Ccoef	*SE(Coef)*	*Lower 0.95*	*Upper 0.95*	*Chisq*	*p-value*
(*Intercept*)	5.4162	6.8875	-9.0165	27.9868	0.5393	0.4627
X1	0.3940	0.8974	-2.0224	4.3726	0.1312	0.7172
X2	-0.2739	0.5412	-2.3155	1.4367	0.1575	0.6915
X3	-0.4802	0.1873	-1.7300	-0.1685	12.230	0.0005
X4	-0.0003	0.0002	-0.0015	0.0001	2.1065	0.1467

Inference based on the profile penalized log-likelihood produces stable and well-defined odds ratios. The likelihood ratio test yields a value of 15.6571 with a p-value of 0.0035, while the Wald statistic is 10.4158 with a p-value of 0.0340. Both the likelihood ratio test and the Wald test indicate that the Firth model is statistically significant overall and is able to explain variation in Family Quality Index categories across provinces [8].

From the Firth logistic regression results, the coefficient for X_3 is −0.4802 with a 95% confidence interval (logit scale) of (−1.7300, −0.1685). The corresponding odds ratio is $\exp(-0.4802) \approx 0.62$ with a 95% confidence interval ranging from $\exp(-1.7300) \approx 0.18$ to $\exp(-0.1685) \approx 0.84$. This odds ratio indicates that a one-percentage-point increase in the poverty rate reduces the odds of a province being classified as “responsive” in the Family Quality Index by approximately 38%, assuming other predictors remain constant. The 95% confidence interval does not include 1, indicating that the effect of X_3 is statistically significant, consistently negative, and substantively meaningful rather than a small-sample artifact. Because the events-per-variable (EPV) ratio in this study was relatively low (approximately 1.25), an additional analysis was performed using a simpler model to examine whether the main findings remained consistent. The reduced Firth logistic regression model included only poverty rate (X3), which was identified as the strongest predictor in the full model. The results showed that the poverty rate continued to have a negative and statistically significant effect on the probability of a province being classified as responsive (coef X3 = −0.598, $p < 0.001$). The estimated coefficient was also similar in magnitude to that obtained from the full model. Taken together, these results suggest that poverty remains the most influential factor associated with provincial FQI categories and that the overall conclusion is largely unchanged when the model is simplified.

Model performance comparison in Table 7 shows that the Firth logistic regression yields a lower AIC than the MLE model, along with higher accuracy and balanced accuracy. The Firth model also achieves perfect sensitivity and a higher Kappa value, indicating better model fit under imbalanced data conditions, but only for apparent performance [3].

Table 7. Apparent Classification Performance of Conventional Logistic Regression and Firth Logistic Regression Models

Reference	Model Firth	Model MLE
AIC	11.87876	15.88
Accuracy	0.9706	0.9412
95% CI	(0.8467, 0.9993)	(0.8032, 0.9928)
P-Value [Acc > NIR]	0.03074	0.1054
Kappa	0.8722	0.7655
Sensitivity	1.0000	0.9655
Specificity	0.8000	0.8000

In this case, it is not clearly observed whether the MLE model systematically misclassifies minority provinces due to the small sample size; however, the MLE model incorrectly classifies a province in the dominant category, namely Gorontalo, which has an FQI value of 78.24. In contrast, the Firth model maintains predictions that are more consistent with the observed data, particularly for provinces in eastern Indonesia. Table 8 presents the FQI values, actual categories, and predicted categories from both models.

Table 8. FQI Values, Actual Categories, and Predicted Categories

Province	FQI	Actual	Firth Model	MLE Model
Aceh	78.62	Responsive	Responsive	Responsive
North Sumatra	77.22	Responsive	Responsive	Responsive
West Sumatra	77.77	Responsive	Responsive	Responsive
Riau	77.99	Responsive	Responsive	Responsive
Jambi	77.27	Responsive	Responsive	Responsive
South Sumatra	76.73	Responsive	Responsive	Responsive
Bengkulu	78.39	Responsive	Responsive	Responsive
Lampung	76.49	Responsive	Responsive	Responsive
Bangka Belitung Islands	79.64	Responsive	Responsive	Responsive
Riau Islands	78.69	Responsive	Responsive	Responsive
DKI Jakarta	80.57	Responsive	Responsive	Responsive
West Java	77.09	Responsive	Responsive	Responsive
Central Java	78.74	Responsive	Responsive	Responsive
DI Yogyakarta	80.39	Responsive	Responsive	Responsive
East Java	78.81	Responsive	Responsive	Responsive
Banten	76.05	Responsive	Responsive	Responsive
Bali	81.49	Responsive	Responsive	Responsive
West Nusa Tenggara	**72.95**	**Moderately**	**Responsive**	**Responsive**
East Nusa Tenggara	72.68	Moderately	Moderately	Moderately
West Kalimantan	75.72	Responsive	Responsive	Responsive
Central Kalimantan	76.88	Responsive	Responsive	Responsive

Province	FQI	Actual	Firth Model	MLE Model
South Kalimantan	79.23	Responsive	Responsive	Responsive
East Kalimantan	79.48	Responsive	Responsive	Responsive
North Kalimantan	75.77	Responsive	Responsive	Responsive
North Sulawesi	78.18	Responsive	Responsive	Responsive
Central Sulawesi	75.79	Responsive	Responsive	Responsive
South Sulawesi	79.37	Responsive	Responsive	Responsive
Southeast Sulawesi	77.68	Responsive	Responsive	Responsive
Gorontalo	**78.24**	**Responsive**	**Responsive**	**Moderately**
West Sulawesi	77.98	Responsive	Responsive	Responsive
Maluku	74.08	Moderately	Moderately	Moderately
North Maluku	76.81	Responsive	Responsive	Responsive
West Papua	74.20	Moderately	Moderately	Moderately
Papua	67.12	Moderately	Moderately	Moderately

Table 9. Comparison of LOOCV Classification Performance Between MLE and Firth Logistic Regression Models

Metric	Firth	MLE
Accuracy	0.8823529	0.8529412
Kappa	0.5310345	0.4585987
Sensitivity	0.9310345	0.8965517
Specificity	0.6000000	0.6000000

To assess how well the classification models would perform on new observations, Leave-One-Out Cross Validation (LOOCV) was applied to both the standard logistic regression model estimated by maximum likelihood and the Firth logistic regression model. As shown in Table 9, the Firth model achieved a classification accuracy of 88.24%, slightly higher than the 85.29% obtained by the MLE model. The Firth approach also produced better agreement, reflected by a higher Kappa statistic (0.531 compared to 0.459), and showed greater sensitivity in correctly identifying provinces classified as responsive (0.931 versus 0.897). Both models yielded the same specificity value of 0.600. Overall, these results suggest that the Firth logistic regression model offers a modest but consistent improvement in classification performance, making it a more suitable choice for datasets characterized by a limited number of observations and an imbalanced distribution of categories.

4. DISCUSSIONS

The findings of this study indicate that the empirical characteristics of the Family Quality Index (FQI) data at the provincial level meet classical conditions that challenge maximum likelihood-based logistic regression, namely small sample size and class imbalance in the response variable. This finding is consistent with the applied statistical literature emphasizing the limitations of MLE under sparse and regionally aggregated data conditions [6]. The inability of the MLE model to identify significant predictors, despite a reduction in residual deviance, highlights that overall model significance does not necessarily imply stable parameter inference. The Firth logistic regression approach effectively addresses this issue through first-order bias correction, resulting in more reliable and well-defined parameter estimates [3], [4], [6].

The significant effect of the poverty rate variable indicates that regional economic pressure is a key factor in distinguishing family quality across provinces. Provinces with higher poverty rates have a significantly lower probability of being classified as having a Family Quality Index that is responsive to gender equality and child protection. This finding is consistent with a broad body of empirical literature showing that poverty is a primary determinant of quality of life and family functioning. Studies on Family Quality of Life demonstrate that poverty negatively affects multiple family dimensions, including health, productivity, emotional well-being, and family interactions [15], [16]. Other studies also show that economic pressure directly weakens family resilience, particularly in physical and economic dimensions [17]. In addition, energy poverty and limited economic resources have been shown to significantly reduce family quality of life [18] and to affect home environment quality as well as child development outcomes [19].

Furthermore, cross-country studies consistently indicate that poverty has a causal relationship with declines in family well-being and child development outcomes through mechanisms such as family stress, limited parental investment, and unfavorable social environments [20]–[23]. Poverty reduction interventions have also been shown to improve family functioning, including cohesion and communication among family members [24], [25]. These findings reinforce that poverty serves as a key mechanism explaining variations in family quality across regions. Therefore, the significance of the poverty variable in this study is not only statistically robust but also supported by consistent theoretical and empirical evidence. The non-significance of other variables, such as education, unemployment, and population density, suggests that their effects may be indirect or mediated by more complex socio-economic mechanisms. At the provincial aggregate level, these variables may not adequately capture within-region heterogeneity. This finding aligns with the literature, indicating that macro-level indicators often require more granular or multilevel approaches to fully explain variation in social outcomes [26].

Compared with the conventional maximum likelihood approach, the Firth logistic regression model yielded more stable parameter estimates and generally better classification results. However, these findings should not be taken to mean that the model can accurately identify provinces belonging to the minority category. Another issue that should be acknowledged is the low events-per-variable (EPV) ratio. With five minority-category provinces and four explanatory variables, the EPV was approximately 1.25, considerably lower than the value commonly recommended in the logistic regression literature. Although the Firth method was adopted because it is known to reduce bias in small samples and improve estimation under sparse-data conditions, the limited number of events remains a constraint. Consequently, the classification results reported in this study should be interpreted with appropriate caution, particularly when assessing performance for the minority category. Future studies could strengthen the evidence by incorporating data from multiple years, increasing the number of observations, or exploring additional approaches designed to address severe class imbalance.

5. CONCLUSION

This study examined factors related to provincial Family Quality Index (FQI) categories in Indonesia using data from 34 provinces in 2023. Of the four explanatory variables considered, poverty rate was the only factor that remained statistically significant after applying bias correction. The negative coefficient indicates that provinces with higher levels of poverty tend to have a lower likelihood of being classified in the responsive FQI category. This finding underscores the close relationship between regional economic conditions and family quality outcomes.

Given the relatively small number of observations and the imbalance between FQI categories, Firth logistic regression was used as an alternative to conventional logistic regression. The results showed that the Firth approach generally provided better classification performance, yielding higher accuracy, Kappa, and sensitivity values than the maximum likelihood model. Similar conclusions were obtained from the sensitivity analysis, where poverty continued to show a significant negative effect even under a simpler model specification. This consistency increases confidence that the main findings are not driven by a particular choice of model.

From a practical perspective, the results suggest that socio-economic indicators that are routinely published each year may help provide provisional estimates of provincial FQI categories when official FQI statistics are not yet available. However, several limitations should be acknowledged. The analysis is based on cross-sectional data from a single year, and the events-per-variable ratio remains lower than the level commonly recommended in the literature. For this reason, the findings should be interpreted with appropriate caution. Future studies would benefit from incorporating data from multiple years and a larger number of observations to further evaluate the stability and wider applicability of the proposed approach.

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